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Comparative study of design methods for two-way reinforced concrete slab systems: an engineering report in civil engineering.

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COMPARITIVE STUDY OF DESIGN METHODS FOR TWO-WAY REINFORCED CONCRETE SLAB SYSTEMS

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COMPARITIVE STUDY OF DESIGN METHODS FOR TWO-WAY REINFORCED CONCRETE SLAB SYSTEMS

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Civil Engineering

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ABSTRACT

Four different methods, the Equivalent Frame, Strip, Finite Element, and Yield Line Methods, are used to design a two-way concrete slab with an irregular column layout to investigate the relative merits of the methods used. The methods are compared based on complexity of design and calculated design moments. Based on the results, the Yield Line Method is considered to be the superior method for designing this type of slab.

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Chapter 1

INTRODUCTION

1.1 Background

The first reinforced concrete slab was built by C. A. P. Turner in 1906. This first slab design, which was used for a Minneapolis project, had to be load tested to satisfy the building code because no one had experience in the design of concrete slabs [1]. A concrete slab is essentially a large plate, which requires a complex analysis according to classical elastic plate theory. Most designers prefer not to perform such an analysis because of the mathematical complexity involved. In fact, a closed form solution is very rare in plate theory. As a result, designers tried to develop simpler forms of analysis which provide reasonable results.

Since 1906, numerous studies have been done on two-way reinforced concrete slabs to find easier methods of analysis. These studies indicate that there are essentially two broad categories of two-way slab design. The first category is the elastic design approach. This approach is used to compute the moments in the slab based on the elastic distribution of moments within the slab. The second category is the ultimate strength, or plastic, approach. Using this

approach, the designer proportions the moments in the slab based on an assumed ultimate moment distribution. Because of the different assumptions inherent in the various methods, slab moments and steel reinforcement will be different depending on the method of analysis the designer chooses.

The problem for the designer is to decide which method is most appropriate for a given slab. He must determine whether the chosen method satisfies strength and serviceability requirements and whether the solution is economical from both a design and construction point of view.

Although two-way slabs are now widely used in building construction, there is still disagreement over what methods of design are acceptable. Section 13.3.1 of the American Concrete Institute's Building Code Requirements for Reinforced Concrete (ACI Code) states a slab can be designed by any method which satisfies both strength and serviceability [2]. However, the ACI Code provides details for only two design methods. This has led some designers to believe they must use one of the methods detailed in the ACI Code.

1.2 Objectives

The purpose of this report is to examine differ-

ent methods of two-way slab design and compare the results to assess the relative merits of design methods. Specifically, four methods will be used to design a slab with an irregular column layout. The methods will then be compared on the relative ease of design and the acceptability of the design moments.

The first criterion is rather subjective; however, a good comparison can be made based on the amount of work required to compute moments for each method. The complex part of the design is to determine the moments at critical sections, and moment distribution across a particular strip. Once the moments are found, detailing reinforcing steel is straightforward.

The second basis of comparison is the actual computed moments at the critical sections for each method. In some methods, determining these moments is relatively easy; however, they may not be as accurate as the moments computed by a more detailed analysis.

The four methods chosen, Equivalent Frame, Strip, Yield Line, and Finite Element Methods, were selected because they represent some of the most popular design methods. The Equivalent Frame Method, and the Finite Element Method as used in this study, are based on an elastic analysis approach. The other two methods, the Strip, and Yield Line Methods, are based on a plastic, or ultimate strength, analysis. Because the approaches

used with each method differ, the distribution of computed moments differs also.

1.3 Design Slab

Figure 1.1 depicts the key plan for design slab layout. Figure 1.2 details a typical interior bent for the slab. This particular arrangement was chosen for three reasons. First, the slab conformed to the limitations set by the ACI Code for the Equivalent Frame Method. Secondly, the columns were arranged in a somewhat irregular manner. The interior columns were designed to carry the large moment in the X, or transverse, direction, while the exterior columns were designed to resist Y, or longitudinal, direction moments. Thirdly, the interior span was twice as large as the two exterior spans. The large interior span was chosen so the slab would test the limitations of the Equivalent Frame Method, and provide unbalanced moments at the columns so shear and moment transfer at the columns would be significant.

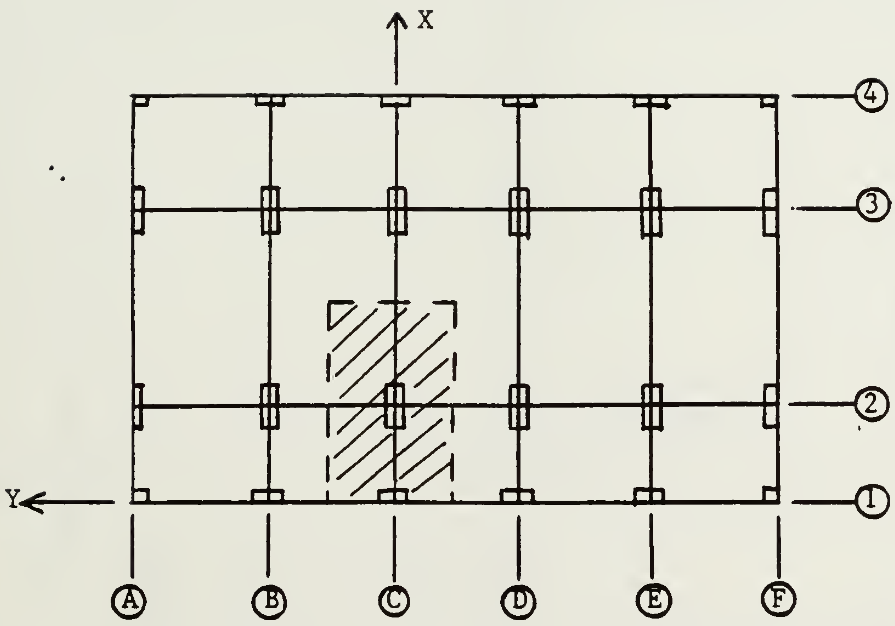


Figure 1.1 Key Plan for Design Slab

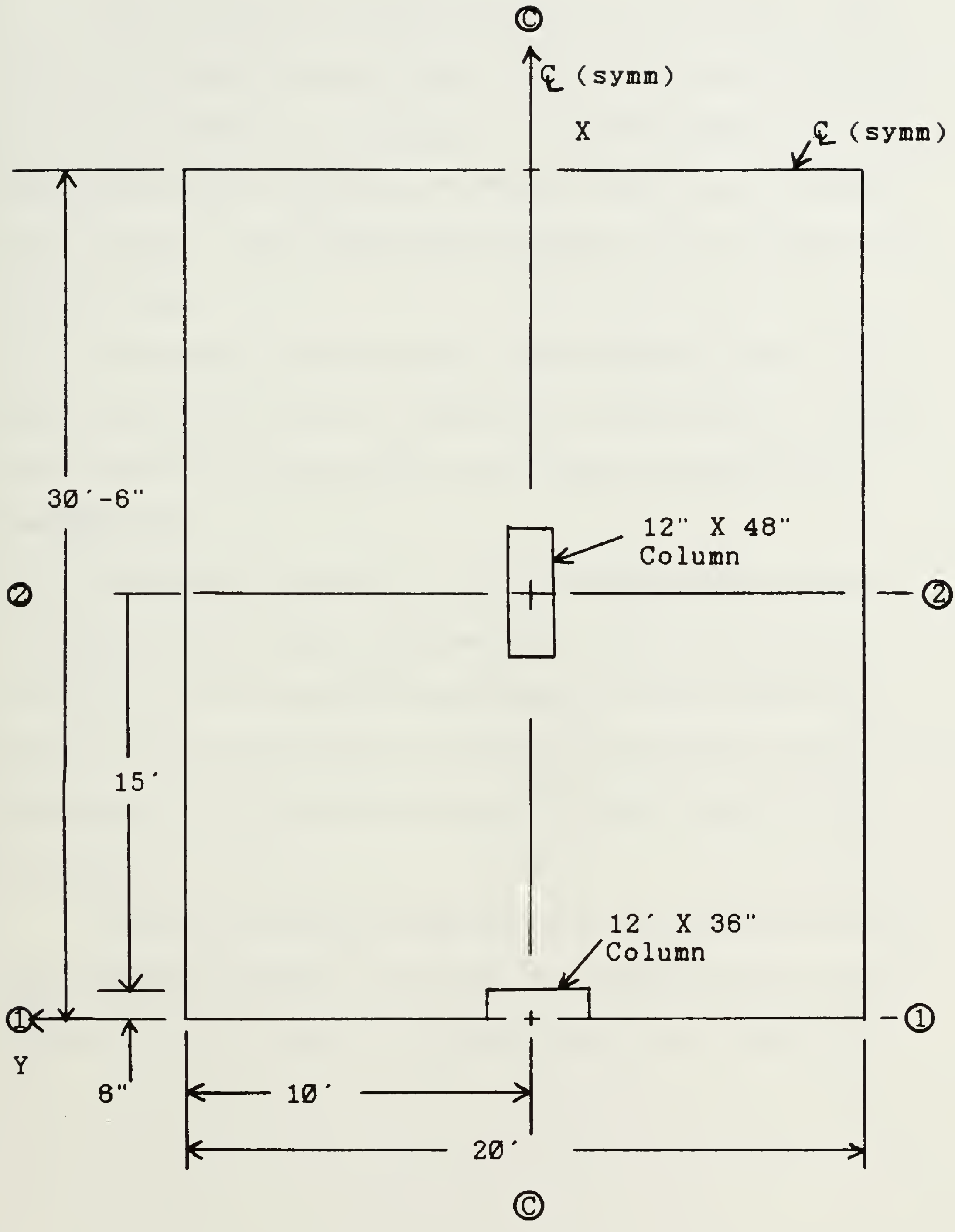


Figure 1.2 Typical Interior Bent for Design Slab

The slab was loaded with typical building loads. The self weight of the slab, based on a slab thickness of 9-1/2 inches, was 119 psf. A superimposed dead load of 30 psf and live load of 50 psf were also included. As a result, the calculated ultimate load of the slab was 294 psf.

The story height used in the analysis was 12 ft. The specified concrete compressive strength was 4,000 psi, and the reinforcing steel yield strength was 60,000 psi.

The slab thickness of 9-1/2 inches was selected in accordance with the requirements of ACI 9.5.3 [2]. Since the slab satisfies minimum thickness requirements, serviceability in terms of deflection control is deemed to be satisfied and deflections need not be computed.

In the following chapters, a brief description of each method is given followed by presentation of the results in the form of bending moment distribution.

CHAPTER 2

DESIGN BY EQUIVALENT FRAME METHOD

2.1 Overview

The Equivalent Frame Method was first implemented in the 1971 ACI Code. Its introduction into the Code was the culmination of many tests and analytical studies done during the early 1960's [1]. This method of design represents the three-dimensional slab system as a number of two-dimensional frames [2]. Each frame is then analyzed to find the moments at the critical sections. Once, these moments are calculated, they are distributed between column and middle strips in accordance with the guidelines given in the Code. The distributed moments are then used to calculate the necessary steel reinforcement.

The three equivalent frames for the slab design are shown in Figures 2.1, 2.2, and 2.3. These figures provide the dimensions for the transverse, longitudinal, and edge equivalent frames, respectively. The Equivalent Frame Method is detailed in Section 13.7 of the ACI Code.

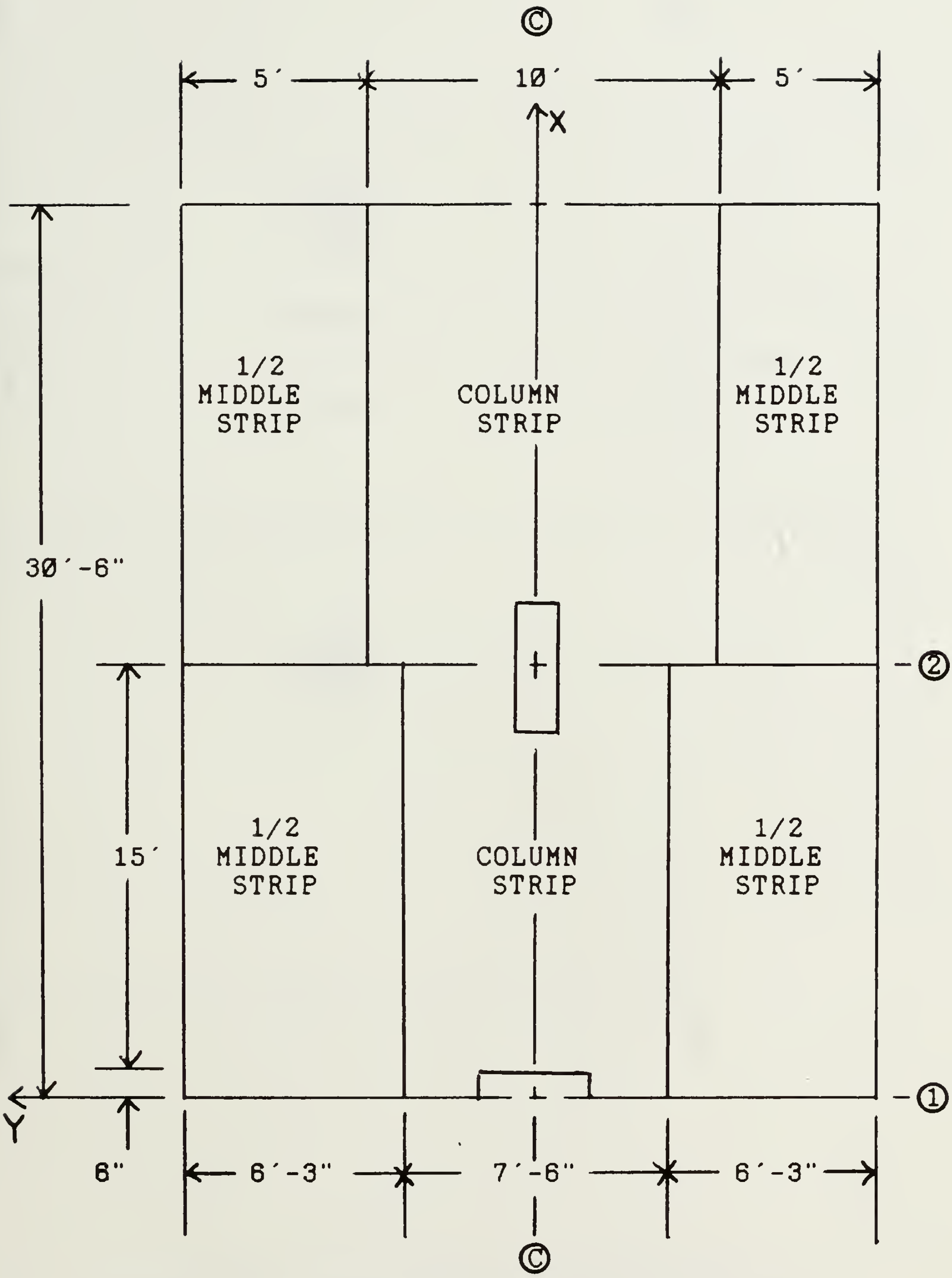


Figure 2.1 Transverse Equivalent Frame

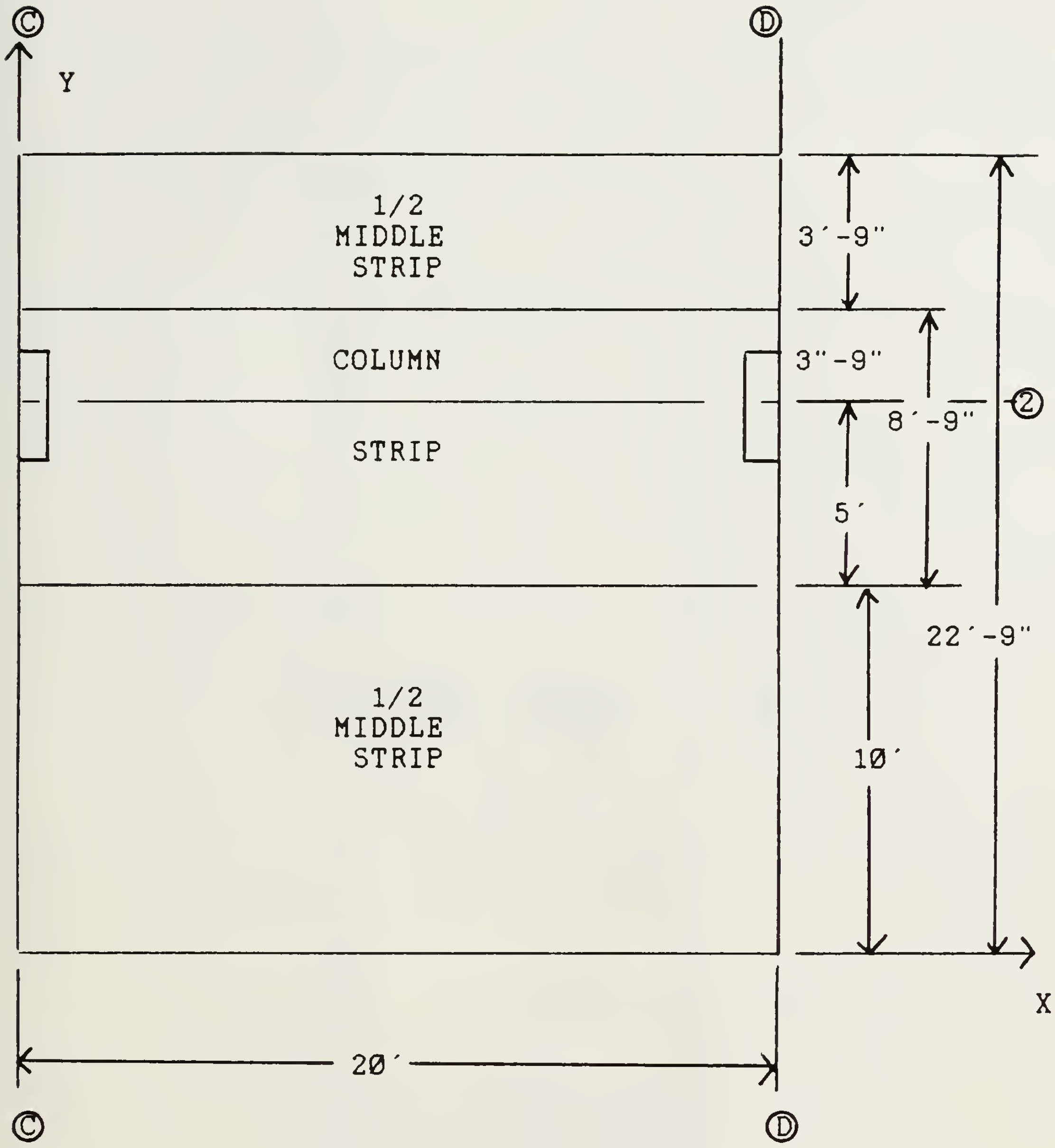


Figure 2.2 Longitudinal Equivalent Frame

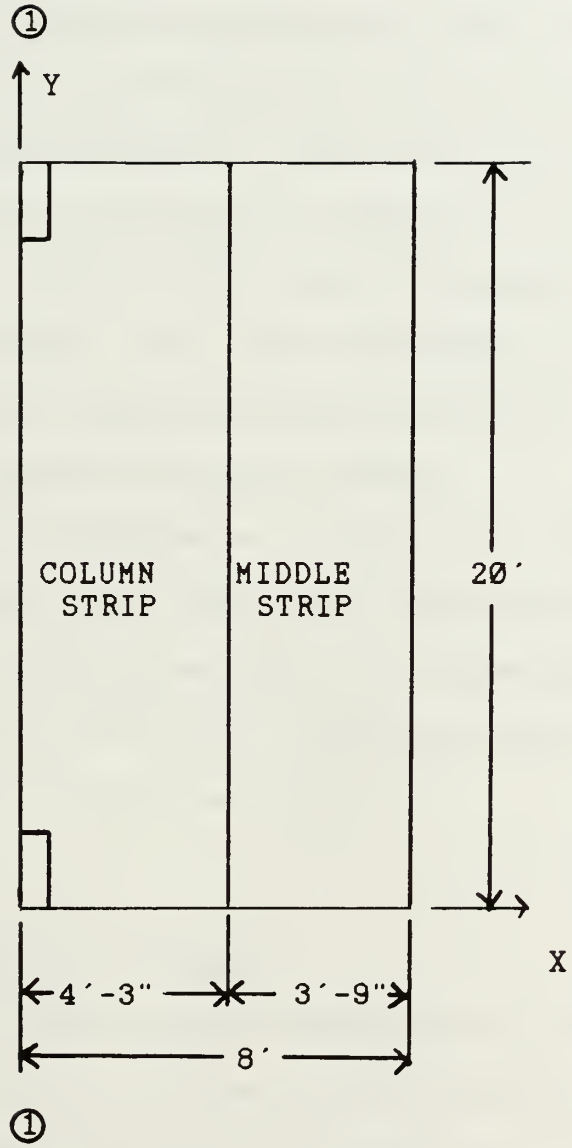


Figure 2.3 Edge Equivalent Frame

2.2 Moment Calculation

The first step in determining the moments in each frame was to compute the column and slab stiffnesses, fixed end moments, and carryover factors. For this study, the slab stiffness, fixed end moments and carry-over factors were computed per Table 13-1 of the 1977 ACI Code [3]. The ACI Code uses the concept of an "equivalent column" [2]. The equivalent column was composed of the columns above and below the slab as well as an attached torsional member.

Once the stiffnesses, fixed end moments, and carryover factors were computed, moment distribution was used to calculate the slab and column moments. The moments were then plotted to determine the total design moments at the critical sections.

2.3 Distribution of Moment

After the total design moments were calculated, they were distributed between the column and middle strips according to ACI 13.6.4 and 13.6.5. These two sections indicate how to distribute the moments based on relative span lengths and relative beam and slab stiffnesses. The column strip at the exterior column was proportioned to resist 99.5% of the total negative moment. The column strip for the interior columns were

designed to resist 75% of the total interior negative moments. All of the column strips were proportioned to resist 60% of the total positive moment. The middle strips were designed to resist those moments not taken by the column strips.

Chapter 3

DESIGN BY FINITE ELEMENT METHOD

3.1 Overview

The Finite Element Method is based on modeling a structure by dividing the structure into small elements. For this study, the concrete slab was divided into plate and beam elements and analyzed using the SAPIV computer program [4]. Because of symmetry, only the portion of the slab shown in Figure 3.1 was needed to adequately model the slab.

The slab was modeled using 96 plate elements and 4 beam elements. The elements in the regions near the columns were small to ensure the large moment gradients near the columns would be adequately modeled. Larger elements were used in the areas farther away from the columns because moment gradients would be smaller. The larger elements were also chosen to conserve the number of elements, thereby reducing data preparation and computer time. The finite column dimensions were modeled using plate elements within the column area with an assumed thickness of 144 inches. Beam elements were used to model the columns. Each beam element was given the size and geometrical properties of the actual column.

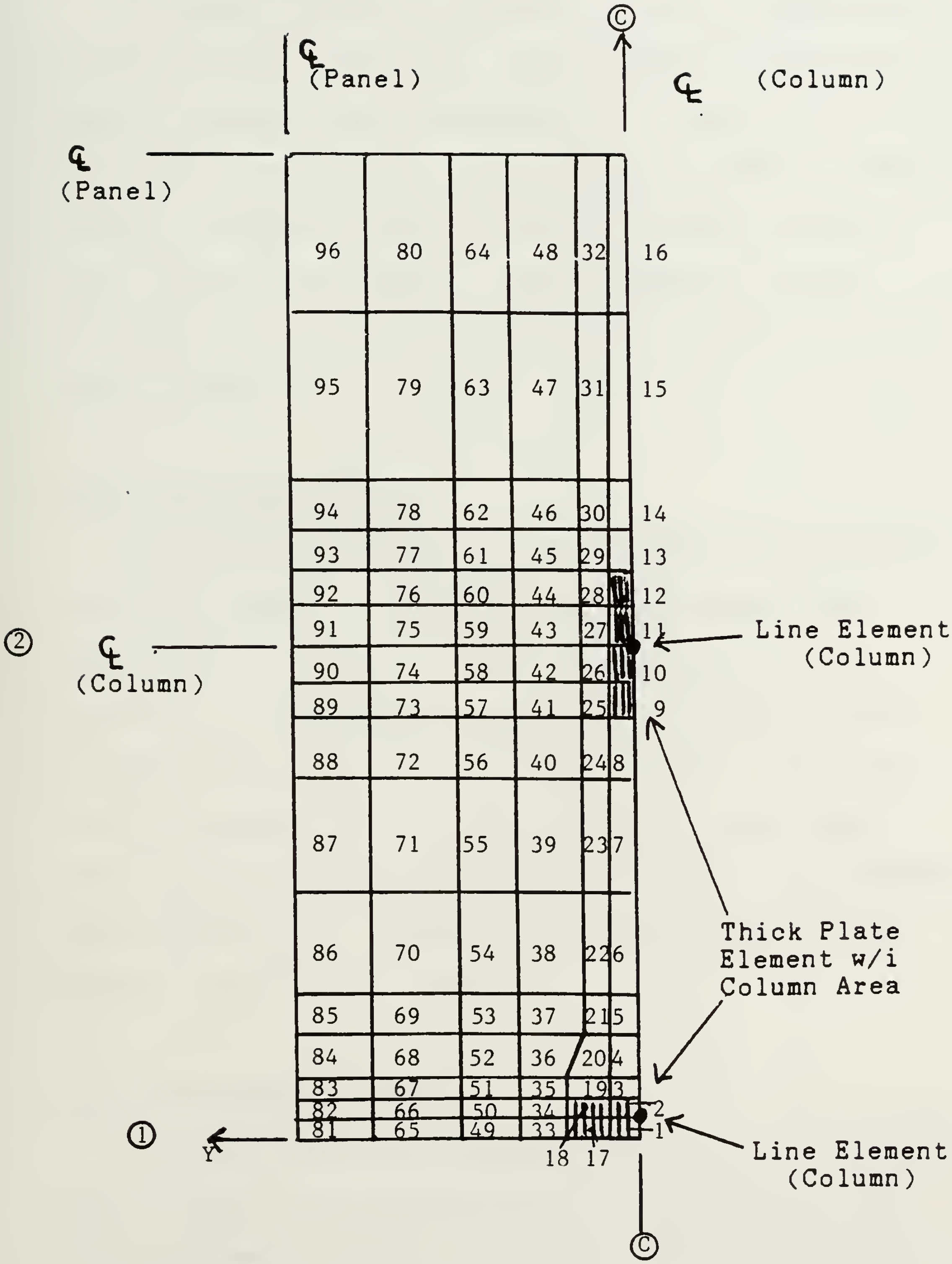


Figure 3.1 Finite Element Grid for Design Slab

3.2 Input Data

The SAPIV computer program requires a large amount of fixed format input data. The program requires the number, location, and deflection and rotational constraints for each node. For each beam element, Young's modulus, Poisson's ratio, geometric properties, and nodal numbers are needed. Nodal numbers, element thickness, and elastic constants are required for each plate element.

3.3 Moment Calculation

The program computes the moments for each element. For plate elements, the moment is computed at the center of the element. The total moment for each element was computed by multiplying the output moment by the element's length in the appropriate direction. The total moment for a particular section was then calculated by summing the total moments of each element along the section. Column moments were given by the moments in the beam elements.

3.4 Distribution of Moment

The distribution of moments between the column and middle strips was straightforward. The moments were found by simply summing the element moments within a particular strip.

Chapter 4

DESIGN BY STRIP METHOD

4.1 Overview

The Strip Method is a lower bound plastic method of design. This method is structured to ensure that the yield criterion is nowhere exceeded in the slab. The method neglects the effect of torsional moments within the slab. If no torsional moments are present, the slab can be divided into strips where only bending moments need to be considered. The designer must divide the slab into strips which provide the applied load a path to the columns. The designer is free to choose his own arrangement; however, his arrangement should closely match the actual load path or large cracks may occur [1].

The strip distribution for the interior and exterior spans of the slab are shown in Figures 4.1 and 4.2, respectively. The interior panel was divided so that a large portion of the load would be taken by the strong band SB1. A smaller portion of the load was carried by strong band SB2. Strong bands are strips of slabs which have a heavier concentration of steel and act as beams to get the load back to the columns [1]. Strong band SB1 was chosen to be eight feet wide, or a

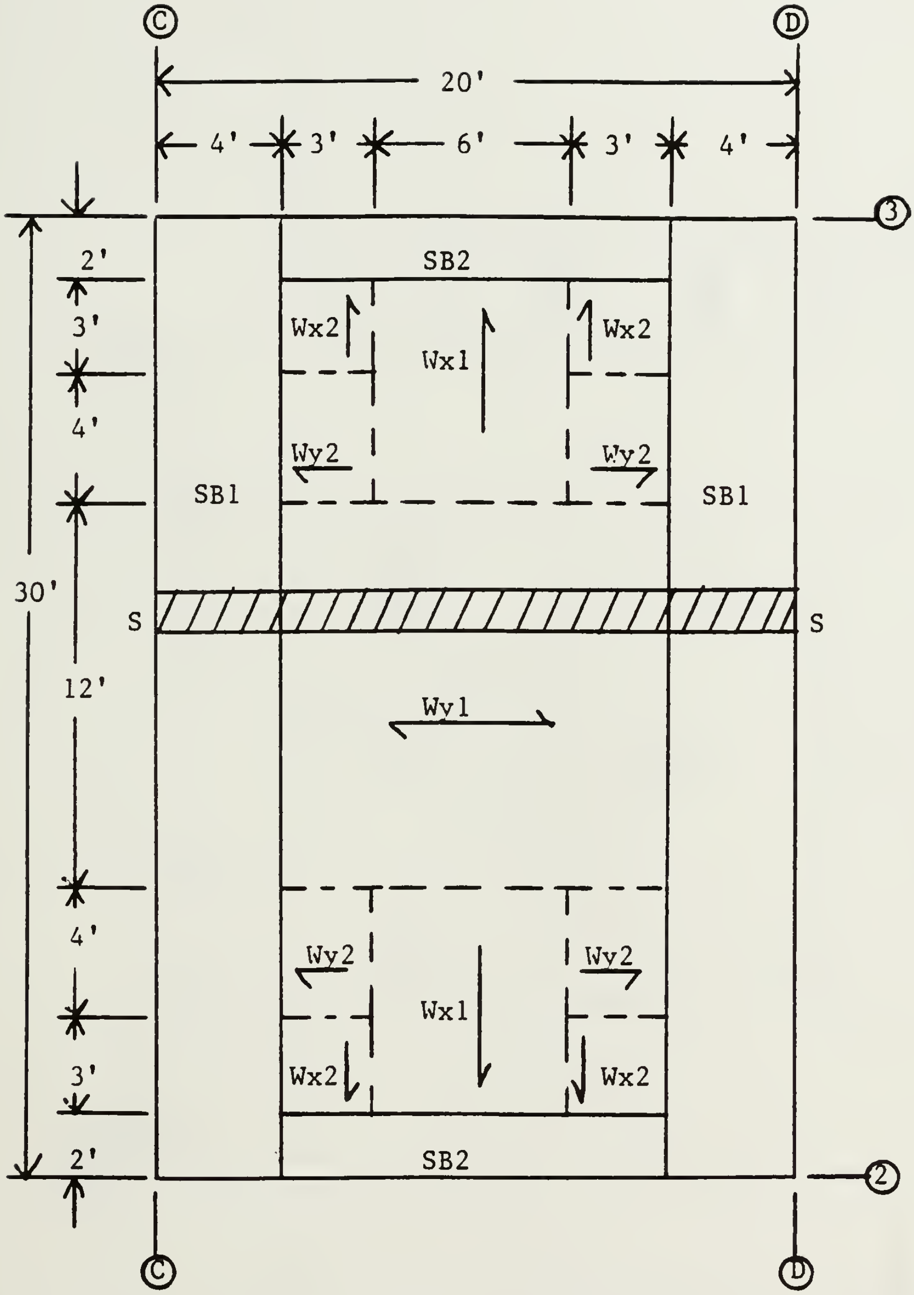


Figure 4.1 Strip Distribution for Interior Panel

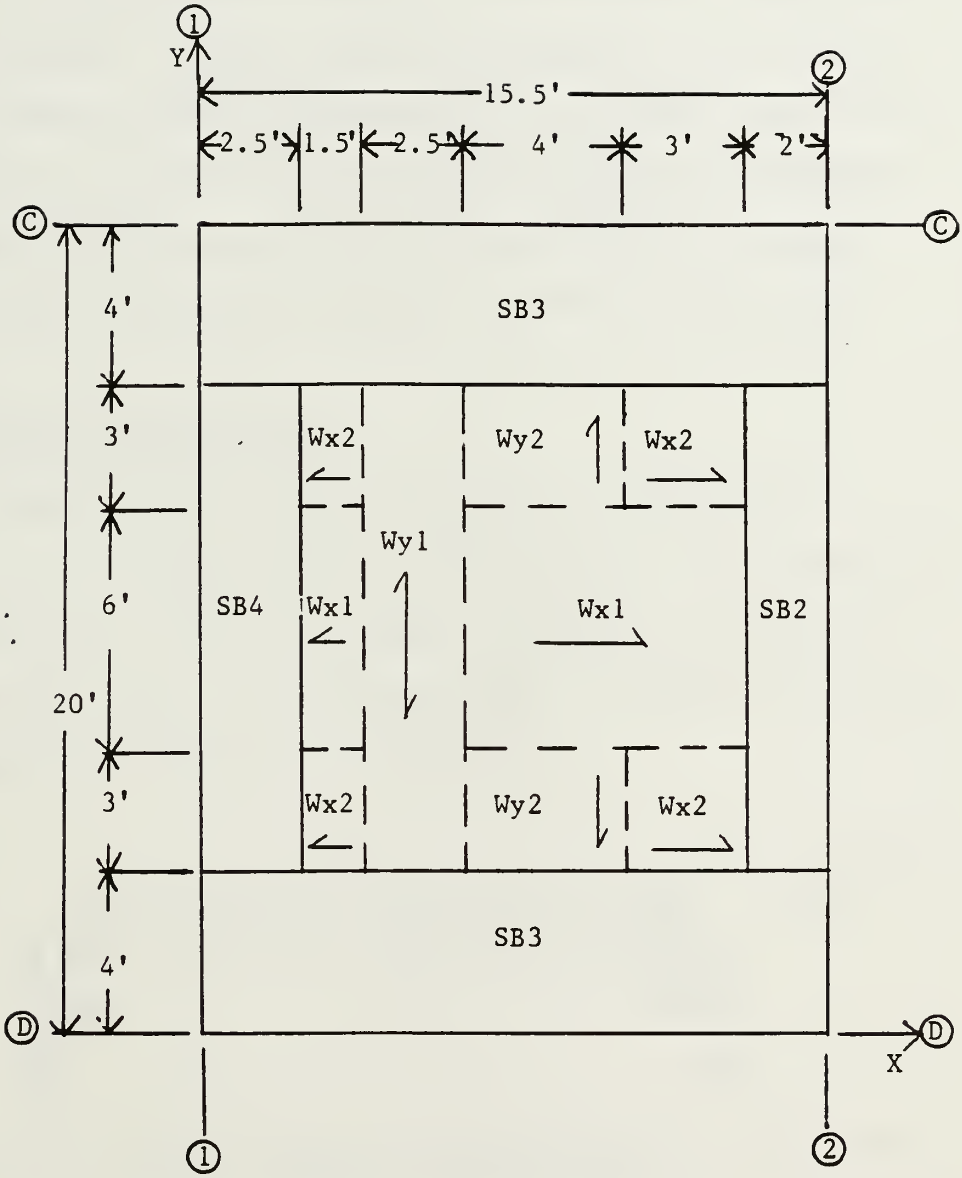


Figure 4.2 Strip Distribution for Exterior Panels

little less than the width of the column strip. Because strong band SB2 carried a much smaller portion of the load, it was chosen to be 4 feet wide.

The strip distribution in the exterior panel was more complex. Because the free edge had very little, or no moment resisting capability, the panel had to be designed so that most of the load would be carried by strong bands SB2 and SB3. Once the strips and load paths were determined, moments could be calculated.

4.2 Moment Calculation

Moments were first calculated for each strip. Each strip was analyzed as a beam fixed at the edges of a strong band with the support reactions distributed along the width of the strong band. A typical loading arrangement is shown in Figure 4.3.

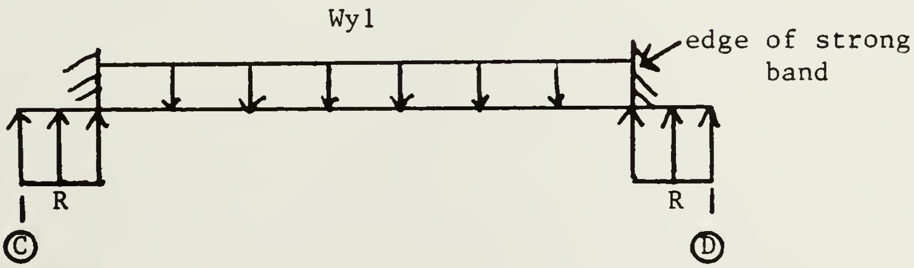


Figure 4.3 Typical Loading Arrangement for Strip SS Shown in Figure 4.1

The strong band moments were determined by moment distribution. The loads on the strong bands were the

calculated loads from the strips and the applied load directly on the strong band itself. The moment distribution factors were the same as those used in the Equivalent Frame Method design.

4.3 Distribution of Moment

Distributing the total moment between the column and middle strips was straightforward. The strong bands were essentially designed to carry the column strip moments, and the simple strips the middle strip moments.

Chapter 5

DESIGN BY YIELD LINE METHOD

5.1 Overview

The Yield Line Method is another ultimate load approach to designing two-way slabs. The method is one of flexural control where it is assumed that once the steel reaches its yield point, load redistribution will occur until all the steel along a line yields and creates a plastic hinge. These plastic hinges, or yield lines, form the basis for the ultimate load design. Yield lines themselves are not straight lines, but rather the center of an area of intense concrete crushing [1]. The designer must predict where the yield lines will form and design the slab accordingly. Unlike the Strip Method, the Yield Line Method is an upper bound method of analysis. That is, unless the designer predicts the actual yield line pattern, he will over predict the strength of the slab. This is not a major concern for most slabs however, because yield line patterns have been published for common slab configurations [1].

For this study, the slab panels were designed primarily to resist a beam, or folding, failure in each direction. The yield line arrangement for a folding

pattern is shown in Figure 5.1. For a particular folding pattern, negative yield lines should develop at the column faces and a positive yield line should develop at mid span. The yield line moments were determined by using the virtual work method of structural analysis.

Once the moments were determined for the folding patterns, the slab was checked to ensure local failure did not occur around the columns. There was a possibility the slab would fail around the column if there was not enough negative steel in the column region. To guard against this local failure, Park and Gamble recommend that the sum of the negative and positive steel reinforcement in the column strip be 1.27 times the overall negative and positive reinforcement.

The slab was also checked to ensure one other potential yield line pattern was not more critical than the two folding yield line patterns. This pattern, shown in Figure 5.2, was checked by using the moments found in the folding pattern to find the ultimate capacity of the slab for this yield line arrangement.

5.2 Moment Calculation

For the yield line patterns shown in Figure 5.1, the ultimate moments were calculated for each direction using the virtual work method. The equations for each

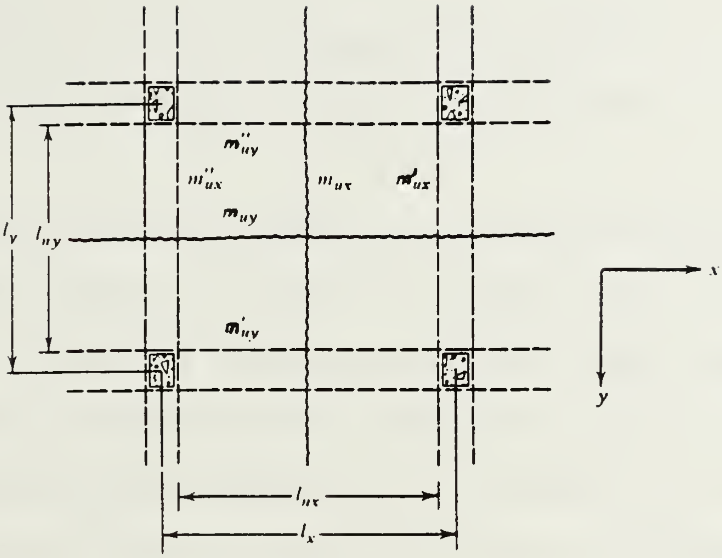


Figure 5.1 Yield Line Pattern for Beamless Slab

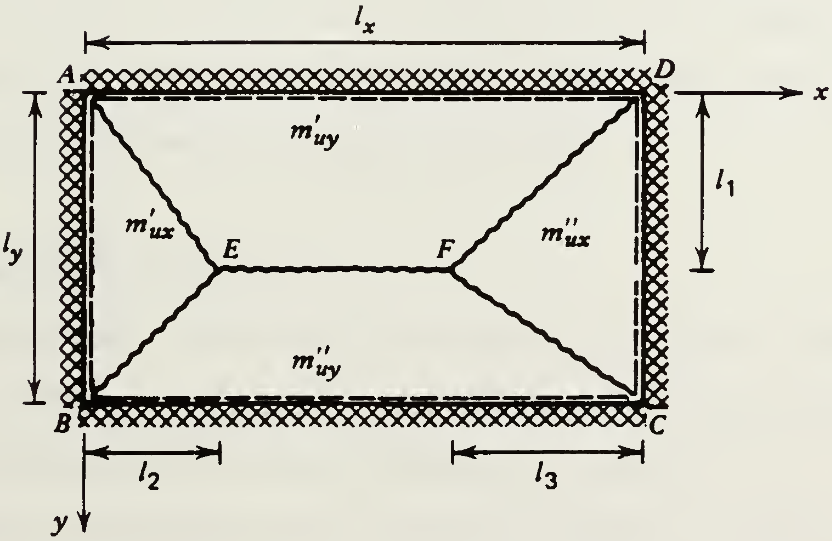


Figure 5.2 Complex Yield Line Pattern

direction were as follows:

$$1/2(m''_{ux} + m'_{ux})l_y + m_{ux}l_y = w_u l_{nx}^2 ly/8$$

$$1/2(m''_{uy} + m'_{uy})l_x + m_{uy}l_x = w_u l_{ny}^2 lx/8.$$

Before using the above equations however, the ratio of the average negative to positive moment had to be assumed. The Comite Europeen du Beton (CEB) recommended that the average negative moment be 1.0 to 1.5 times the average positive moment [1]. A value of 1.5 was chosen for the interior panel in order to maximize the negative moment. For the exterior panel, with a free edge, the CEB recommended the ratio could be as high as 2.0. This higher value was somewhat offset by the assumption of no resisting negative moment at the exterior column. Once these assumptions were made, the two equations could be solved for the average positive and negative moments in each direction.

5.3 Moment Distribution

The moment distribution between the column and middle strips was also per the CEB's recommendations. These recommendations are depicted in Figure 5.3. This particular arrangement was chosen because it would help control potential cracks in the middle strip due to negative moments [1].

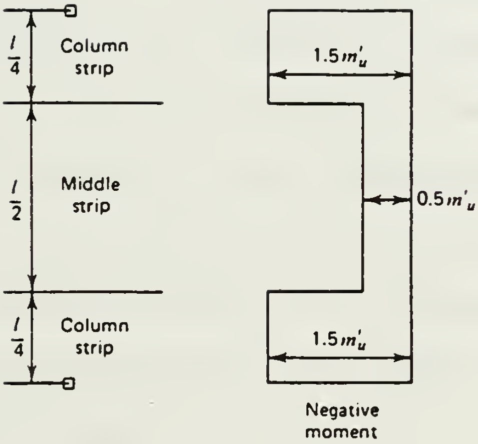


Figure 5.3 Distribution of Negative and Positive Bending Moments

Chapter 6

RESULTS

This chapter presents the design moments computed by each method. Additionally, the distribution of moments computed by the Finite Element Method are presented in order to show the theoretical distribution of elastic moments around the columns. The chapter concludes with a discussion of the results. The discussion focuses on how the computed static moments for each method differ from the actual static moment based on the clear span.

6.1 Design Moments

The design moments and bending moment diagrams for each frame are discussed in the following sections.

6.1.1 Transverse Frame

The distributions of moments for the exterior and interior panels of the transverse frame are shown in Tables 6.1 and 6.2, respectively. The bending moment diagrams for the frame are shown on Figures 6.1 through 6.3.

The moments in the exterior span differed significantly. The negative moments at the exterior column were close to Ø K-FT for the Equivalent Frame and Strip

Table 6.1

DISTRIBUTION OF TOTAL DESIGN MOMENTS FOR TRANSVERSE FRAME -- EXTERIOR SPAN

METHOD	LOCATION	TOTAL (K-FT)	COL STRIP (K-FT)	MID STRIP (K-FT)
EQUIVALENT FRAME	M1	0.0	0.0	0.0
	Mp	46.7	28.0	18.7
	M2	-138.5	-104.0	-34.5
FINITE ELEMENT	M1	-25.0	-35.0	10.0
	Mp	41.6	18.4	23.2
	M2	-95.0	-60.0	-35.0
STRIP	M1	-3.2	-2.0	-1.2
	Mp	39.5	24.2	15.3
	M2	-137.9	-110.3	-27.6
YIELD LINE	M1	-26.0	-16.0	-10.0
	Mp	58.0	32.0	26.0
	M2	-116.0	-87.0	-29.0

M1= moment at exterior column face
Mp= maximum positive moment
M2= moment at interior column face

Table 6.2
DISTRIBUTION OF TOTAL DESIGN MOMENTS FOR TRANSVERSE FRAME -- INTERIOR SPAN

METHOD	LOCATION	TOTAL (K-FT)	COL STRIP (K-FT)	MID STRIP (K-FT)
EQUIVALENT FRAME	Mneg	-265.0	-198.4	-66.6
	Mp	231.6	139.0	92.6
FINITE ELEMENT	Mneg	-280.0	-200.0	-80.0
	Mp	190.0	80.0	110.0
STRIP	Mneg	-264.6	-222.8	-41.8
	Mp	231.4	223.2	8.2
YIELD LINE	Mneg	-298.0	-224.0	-74.0
	Mp	199.0	110.0	89.0

Mneg= negative moment at column face
Mp= maximum positive moment

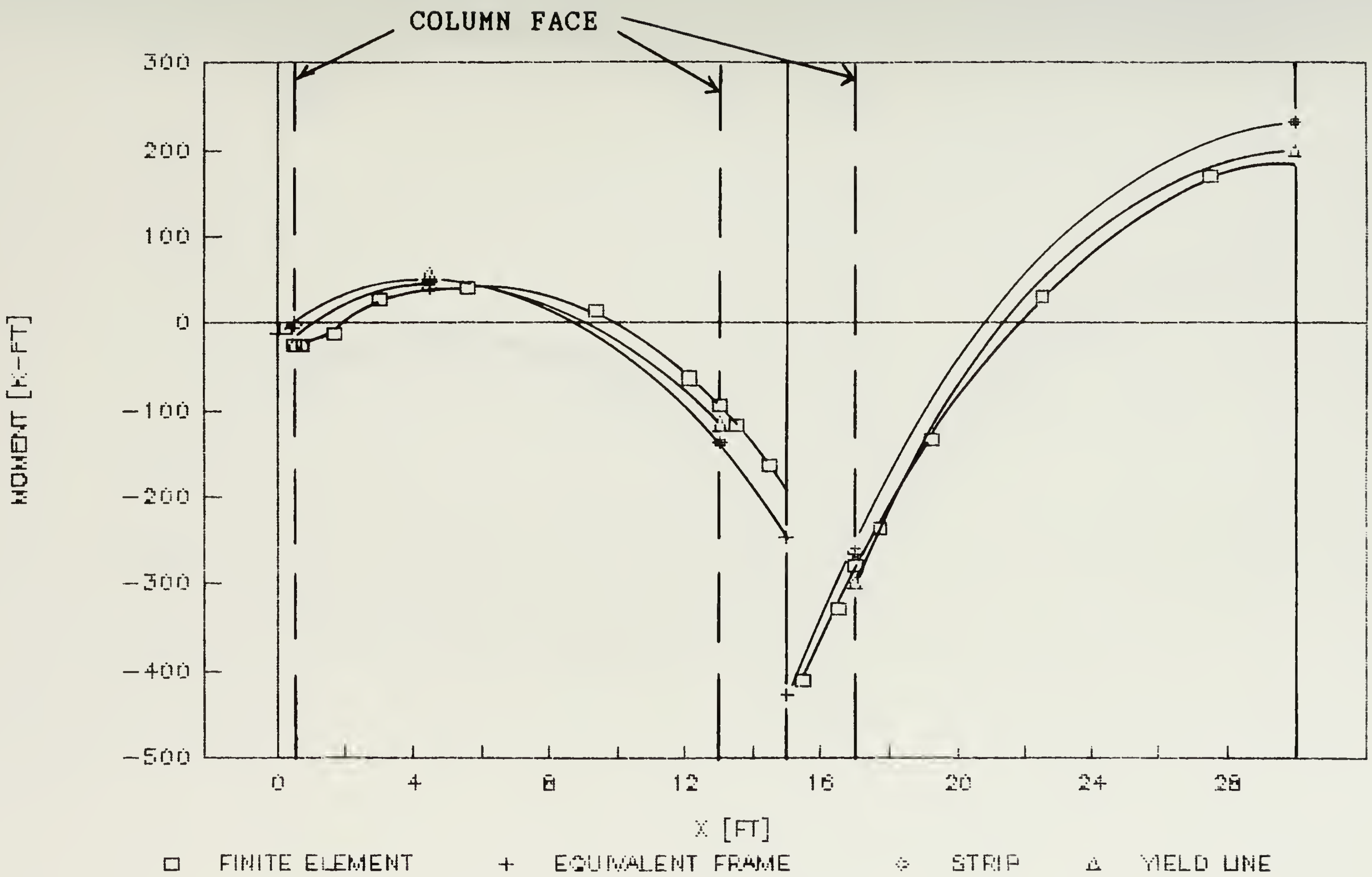


Figure 6.1 Transverse Frame Bending Moment Diagram

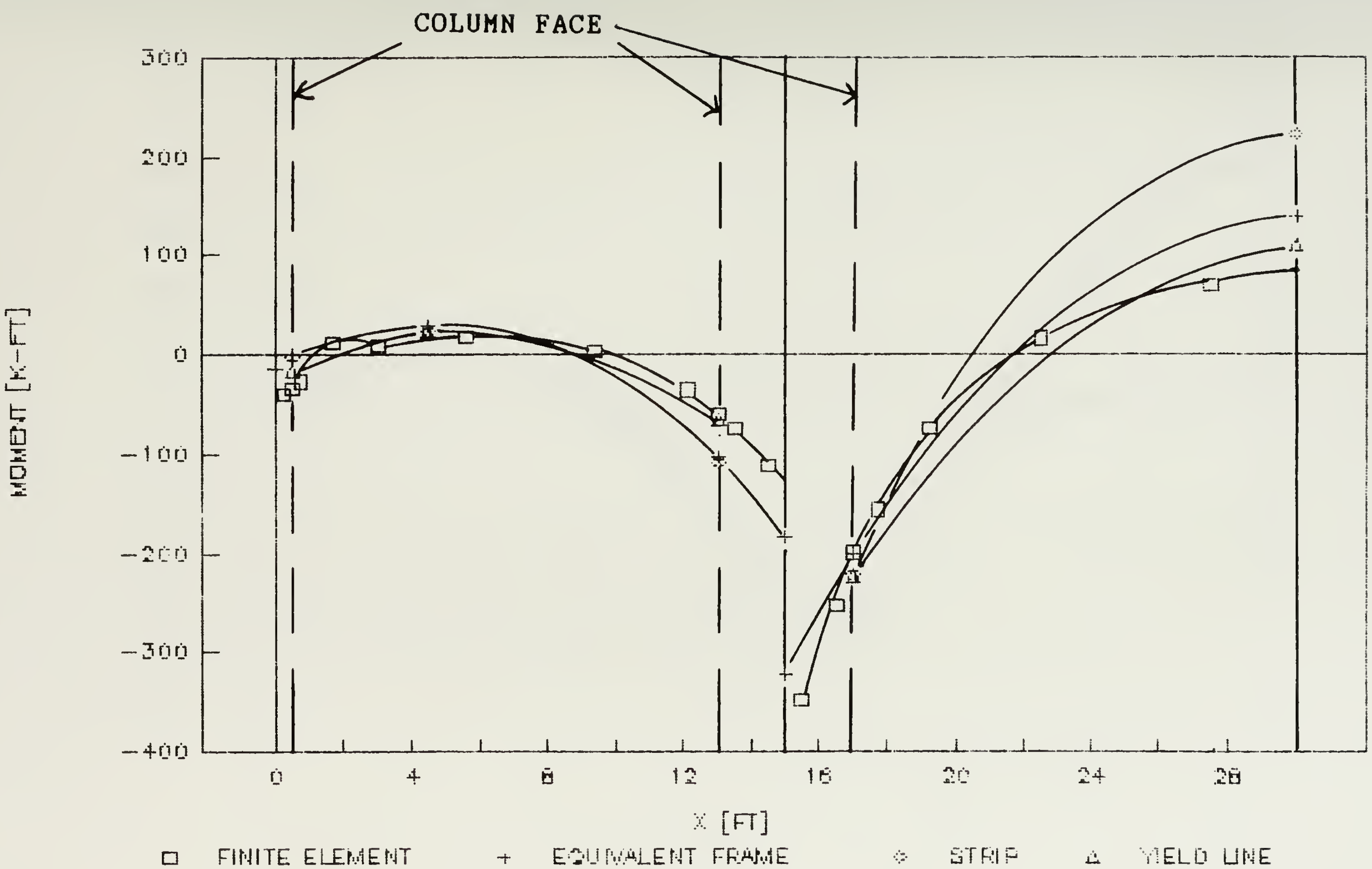


Figure 6.2 Transverse Frame Column Strip Bending Moment Diagram

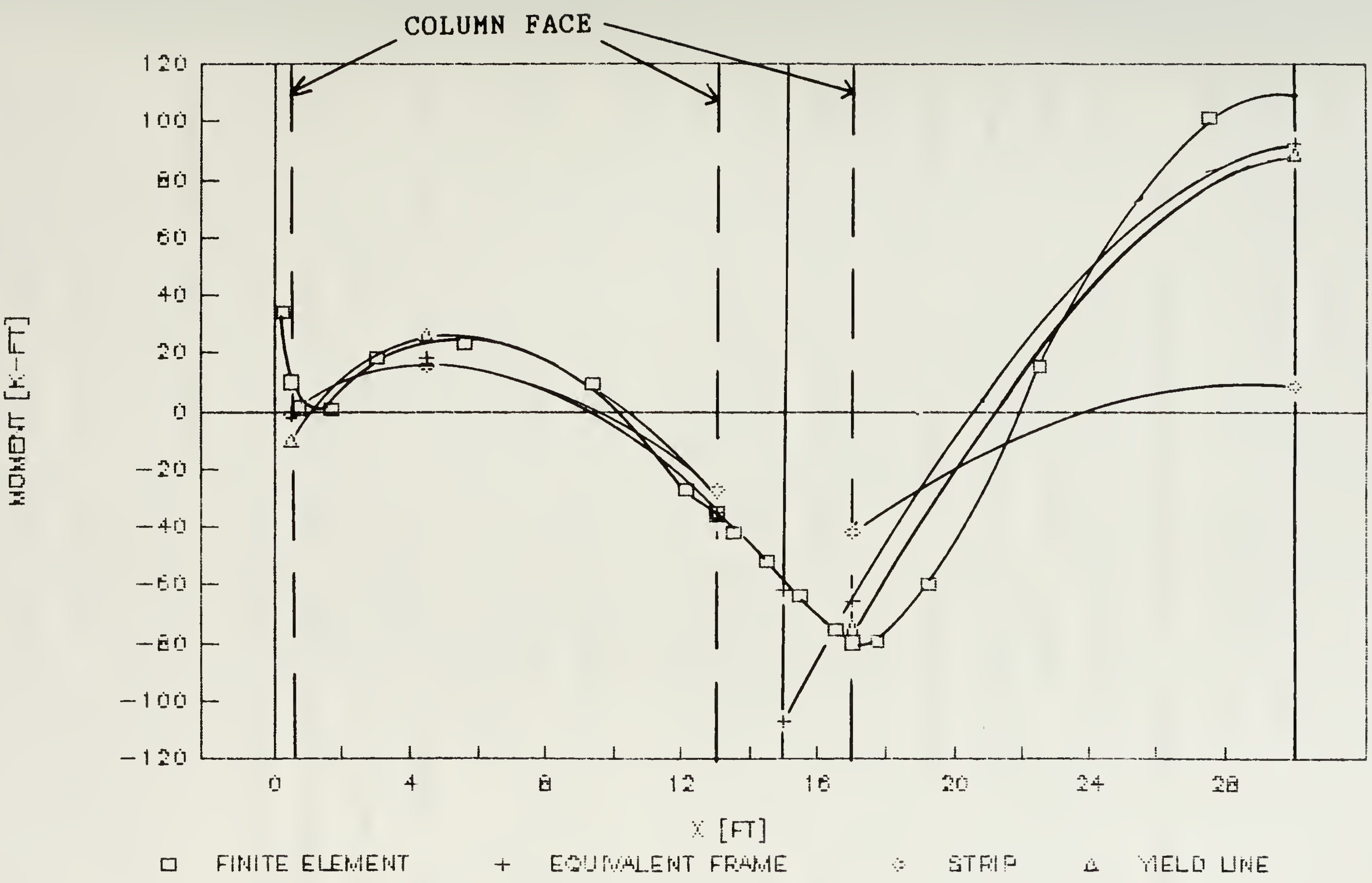


Figure 6.3 Transverse Frame Middle Strip Bending Moment Diagram

Methods and approximately 25 K-FT for the Yield Line and Finite Element Methods. At the interior column, the moments varied from 95 K-FT to 138.5 K-FT. The positive moments ranged from 39.5 to 58.0 K-FT.

The percentage distribution of moments between the column and middle strip was approximately the same except for the Finite Element Method. The Finite Element results indicated a -35 K-FT moment in the column strip. The analysis also produced a +10 K-FT moment in the middle strip where one would typically expect a small negative moment. While this result was unexpected, a review of the input data indicated all data for the elements along this strip were input correctly.

Although there were differences in the interior span, the overall moments were closer. The negative moments varied by approximately 10 percent. The positive moments varied by as much as 18 percent. Generally, the moments computed by the Finite Element and Yield Line Methods had higher negative and smaller positive values than those computed by the Equivalent Frame and Strip Methods.

The distribution of moments for the Finite Element and Strip Methods were noteworthy. As before, the positive middle strip moment computed by the Finite Element Method was higher than the positive column

strip moment. The middle strip moments computed by the Strip Method were only 15 % and 4 % of the total negative and positive moments, respectively.

6.1.2 Longitudinal Frame

The distribution of moments for the longitudinal frame is shown in Table 6.3. Bending moment diagrams are shown in Figures 6.4 through 6.6. Once again, the negative moments computed by the Yield Line and Finite Element Methods were higher than those from the other two methods. The positive moment computed by the Equivalent Frame Method was the highest positive moment. The Strip Method positive moment was only 54% of the Equivalent Frame positive moment.

The positive middle strip moments were higher than the corresponding column strip for both the Yield Line and Finite Element Methods. As before, the Strip Method positive middle strip moment was small compared to the positive column strip moment.

6.1.3 Edge Frame

The design moments for the edge frame are shown in Table 6.4. Figures 6.7 through 6.9 are the bending moment diagrams. For this frame, the highest moments were computed by the Yield Line Method. The negative

Table 6.3

DISTRIBUTION OF TOTAL DESIGN MOMENTS FOR LONGITUDINAL FRAME

METHOD	LOCATION	TOTAL (K-FT)	COL STRIP (K-FT)	MID STRIP (K-FT)
EQUIVALENT FRAME	Mneg	-164.8	-123.6	-41.2
	Mp	136.7	82.0	54.7
FINITE ELEMENT	Mneg	-178.0	-120.0	-58.0
	Mp	105.0	50.0	55.0
STRIP	Mneg	-155.3	-94.4	-60.9
	Mp	73.1	61.0	12.1
YIELD LINE	Mneg	-176.0	-113.0	-63.0
	Mp	114.5	50.0	64.5

Mneg= negative moment at column face
Mp= maximum positive moment

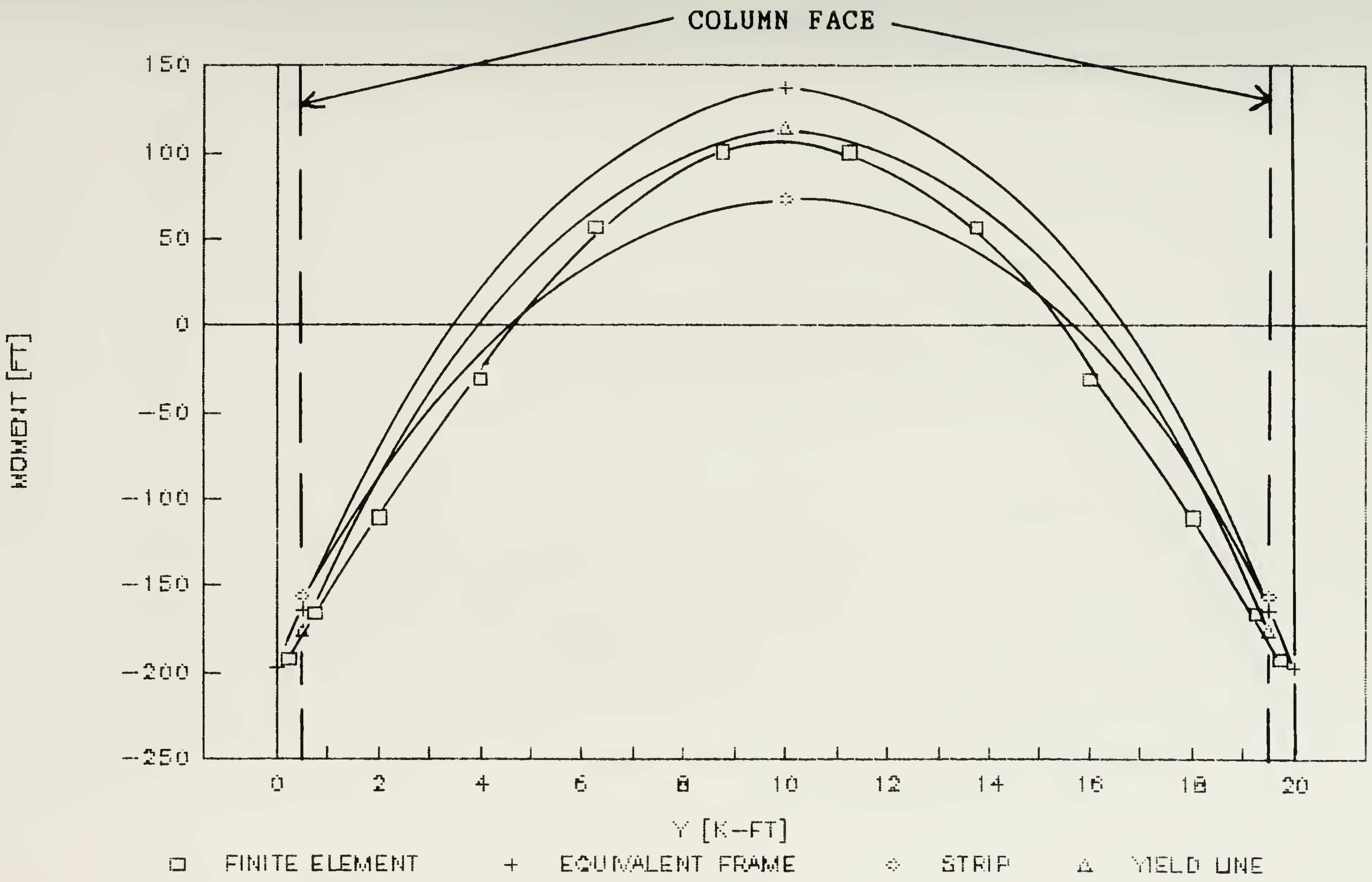


Figure 6.4 Longitudinal Frame Bending Moment Diagram

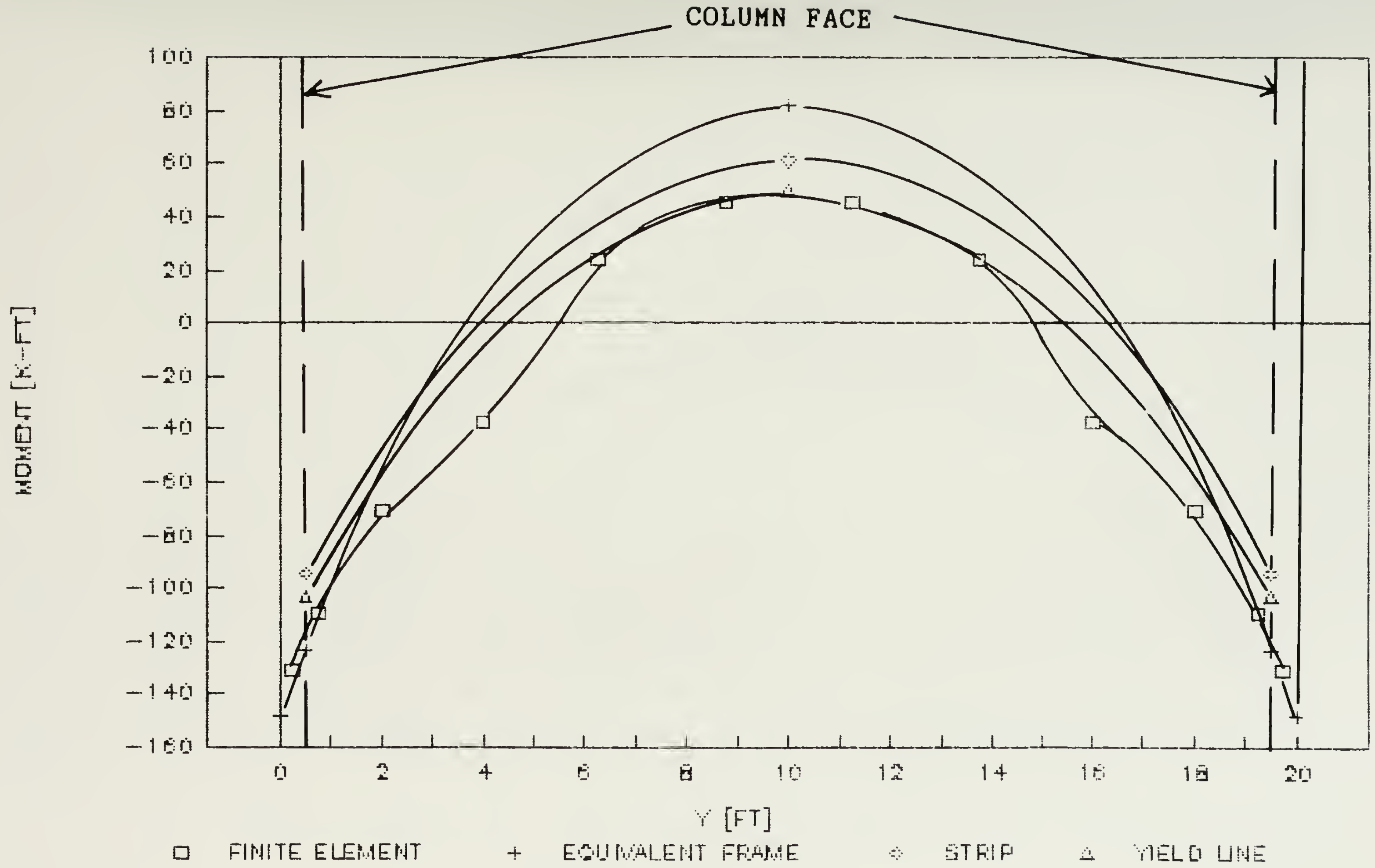


Figure 6.5 Longitudinal Frame Column Strip Bending Moment Diagram

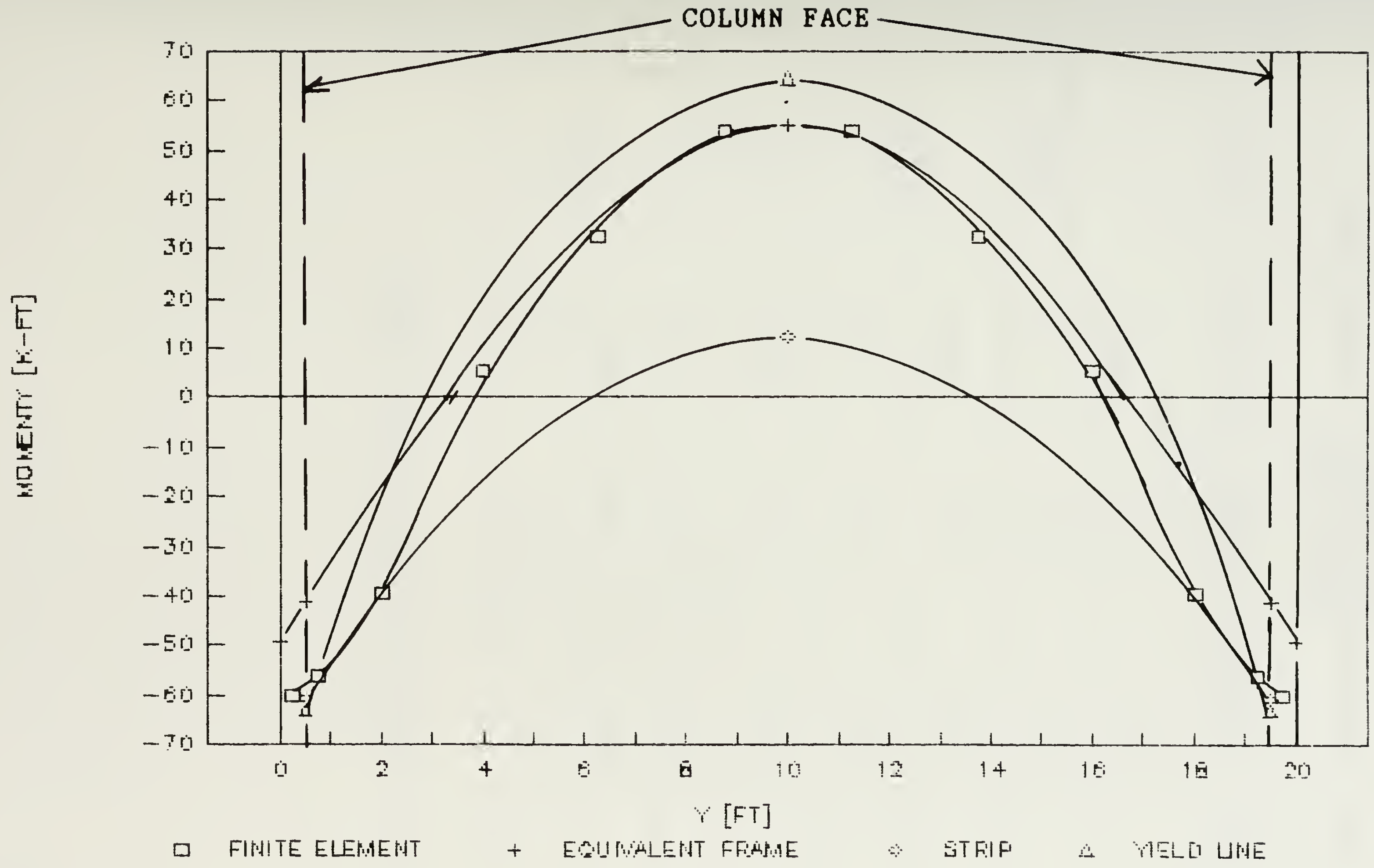


Figure 6.6 Longitudinal Frame Middle Strip Bending Moment Diagram

Table 6.4
DISTRIBUTION OF TOTAL DESIGN MOMENTS FOR
EDGE FRAME

METHOD	LOCATION	TOTAL (K-FT)	COL STRIP (K-FT)	MID STRIP (K-FT)
EQUIVALENT FRAME	Mneg	-48.0	-36.0	-12.0
	Mp	36.6	22.0	14.8
FINITE ELEMENT	Mneg	-45.0	-35.0	-10.0
	Mp	32.0	18.0	14.0
STRIP	Mneg	-55.9	-34.0	-21.9
	Mp	28.5	23.7	4.8
YIELD LINE	Mneg	-60.0	-47.0	-13.0
	Mp	40.0	23.0	17.0

Mneg= negative moment at column face
Mp= maximum positive moment

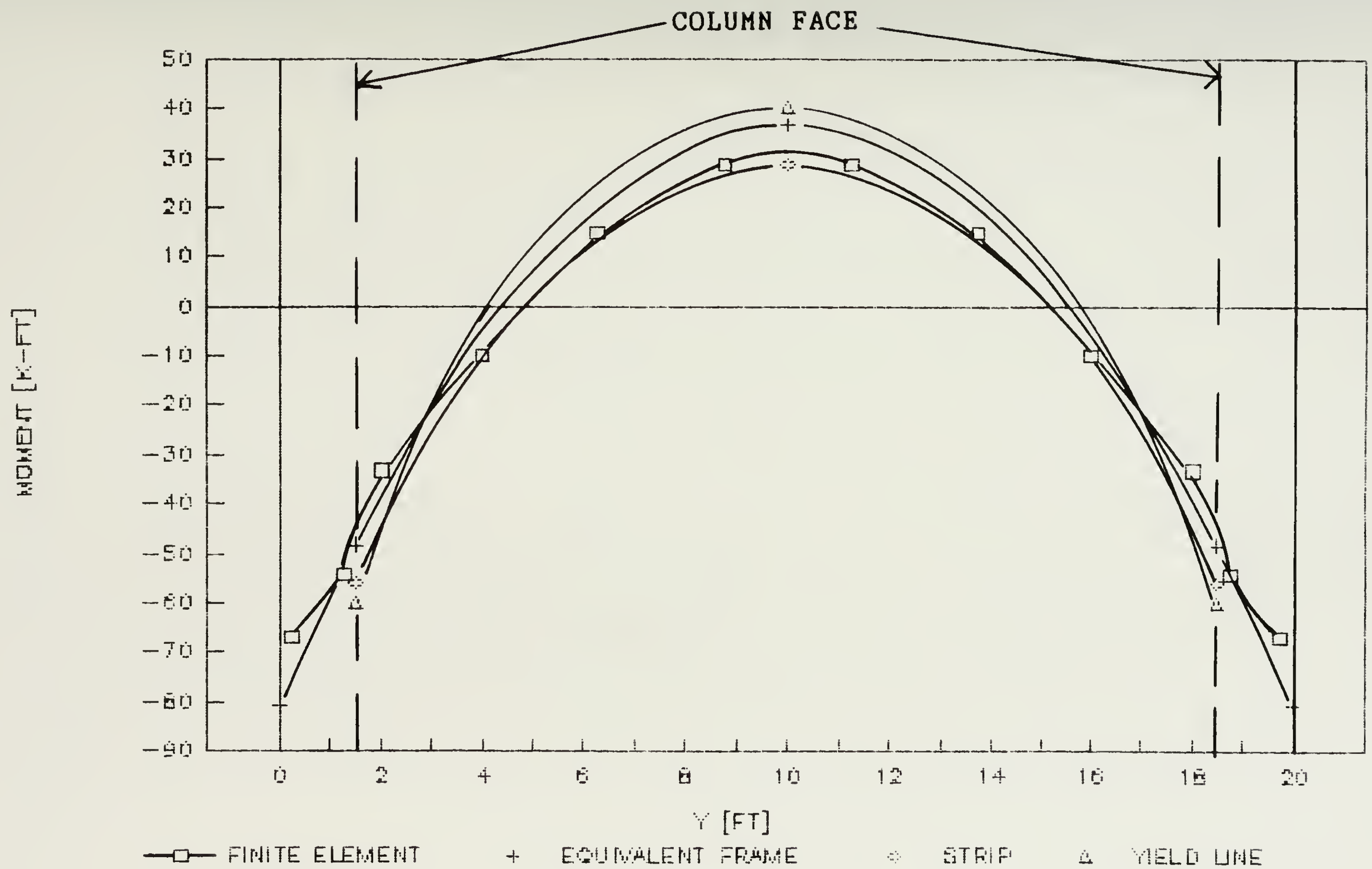


Figure 6.7 Edge Frame Bending Moment Diagram

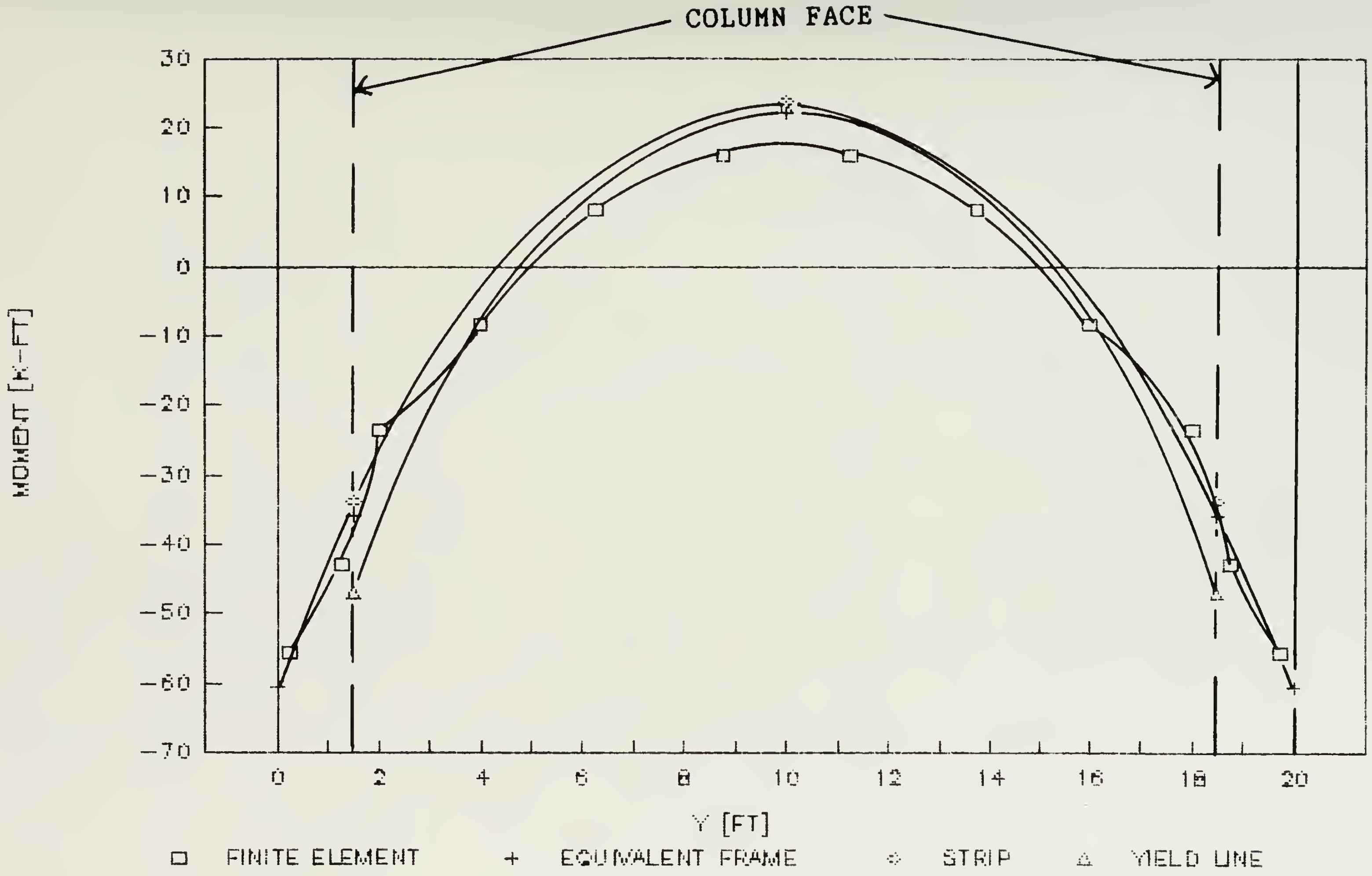


Figure 6.8 Edge Frame Column Strip Bending Moment Diagram

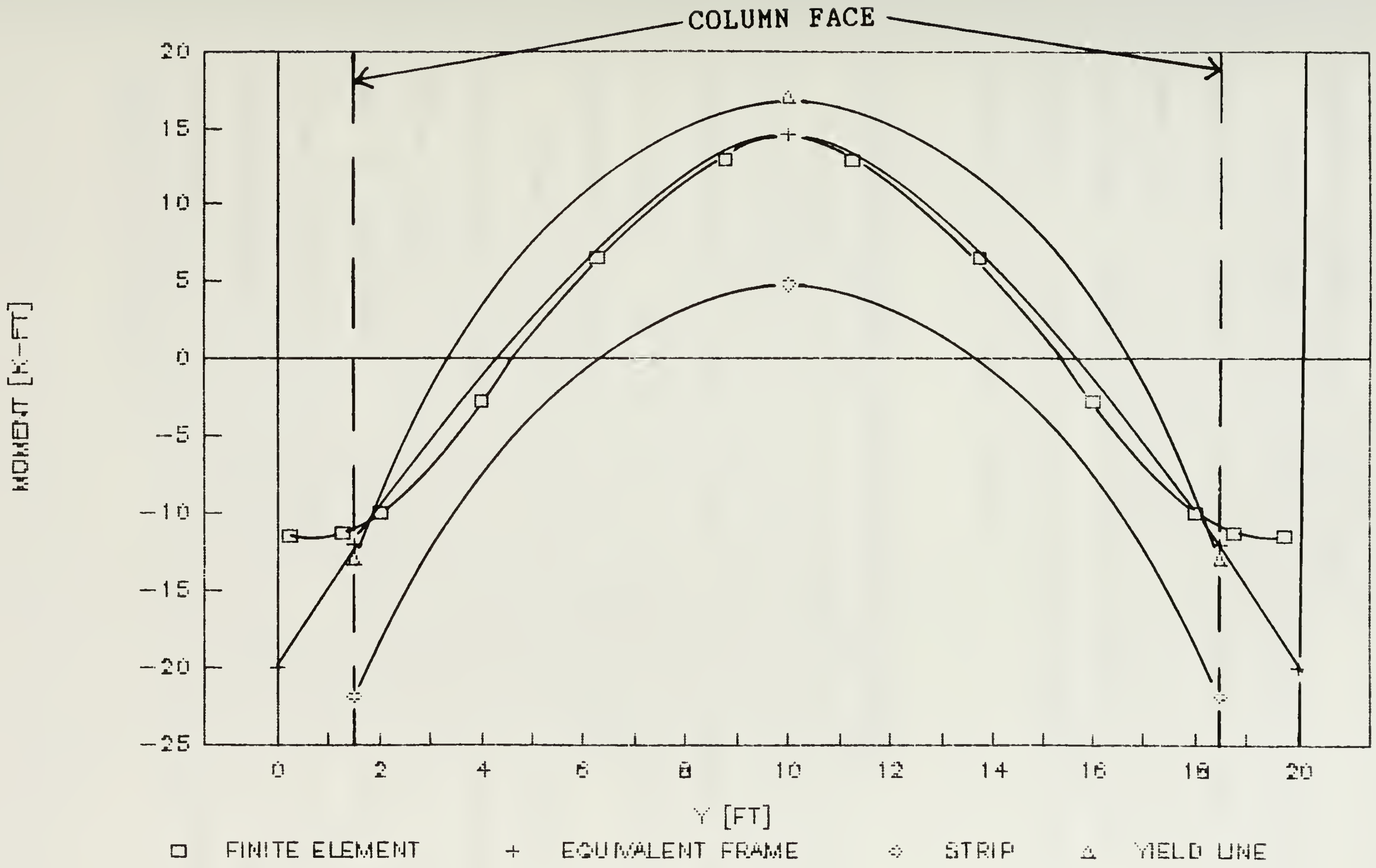


Figure 6.9 Edge Frame Middle Strip Bending Moment Diagram

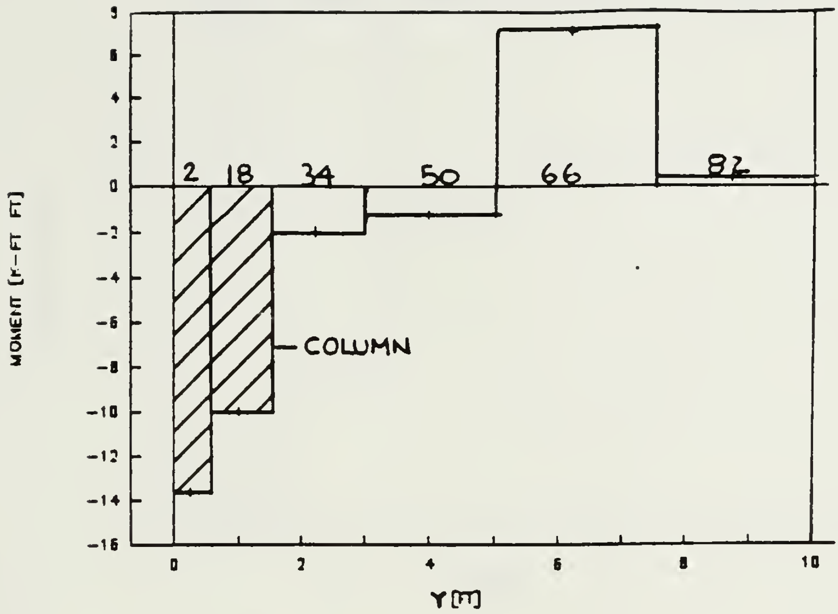
moments computed by the Equivalent Frame and Finite Element Methods were very close.

The column and middle strip bending moment diagrams were similar to the total moment diagram with the exception of the Strip Method. The Strip Method had the lowest negative and highest positive column strip moments. It also had the highest negative and lowest positive middle strip moments.

6.2 Finite Element Distribution of Moment

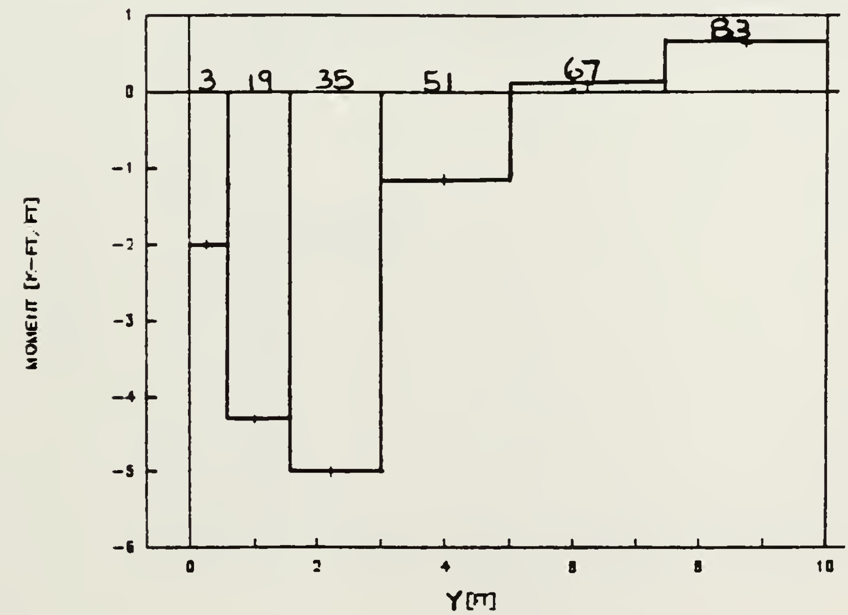
In order to see how the moments were distributed near the columns, plots were made of the moment intensities of each element along a line bordering the face of a column. Figures 6.10 through 6.19 show the moment intensity distribution along these strips.

These plots were significant for two reasons. First, they indicated that in the strip containing the column element, the slab element within the column took most of the moment. The element just outside the column took very little moment. Secondly, for the strips that did not contain the column, the moment intensities were highest near the column and then tapered off gradually.



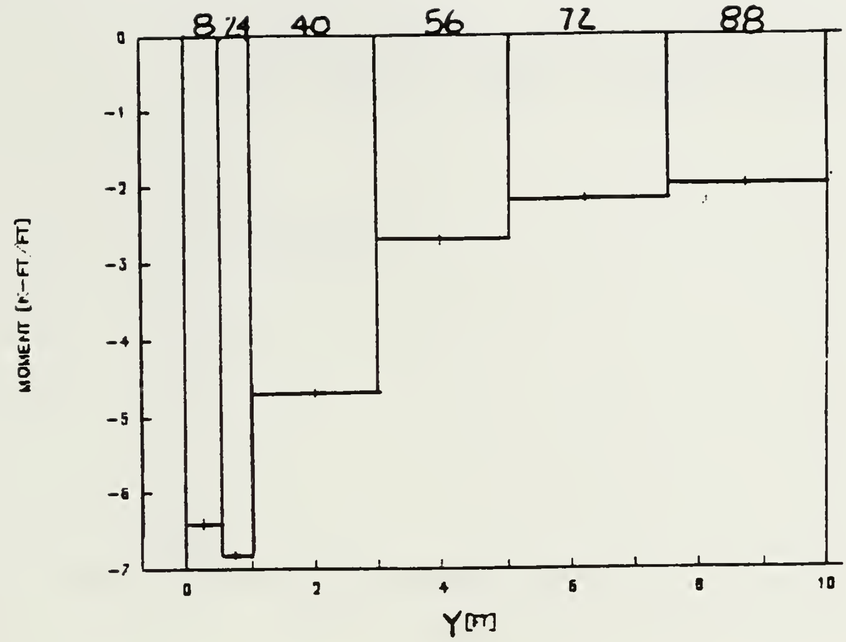
(see Figure 3.1 for element location)

Figure 6.1Ø Mx Moment for Plate Elements 2,18,34,5Ø,66,& 82



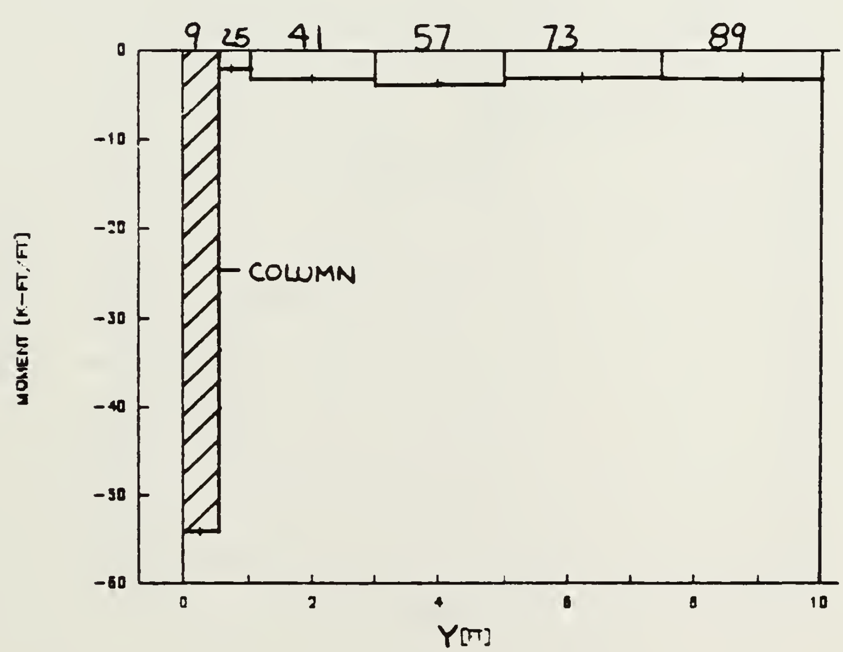
(see Figure 3.1 for element location)

Figure 6.11 Mx Moment for Plate Elements 3,19,35,51,67,& 83



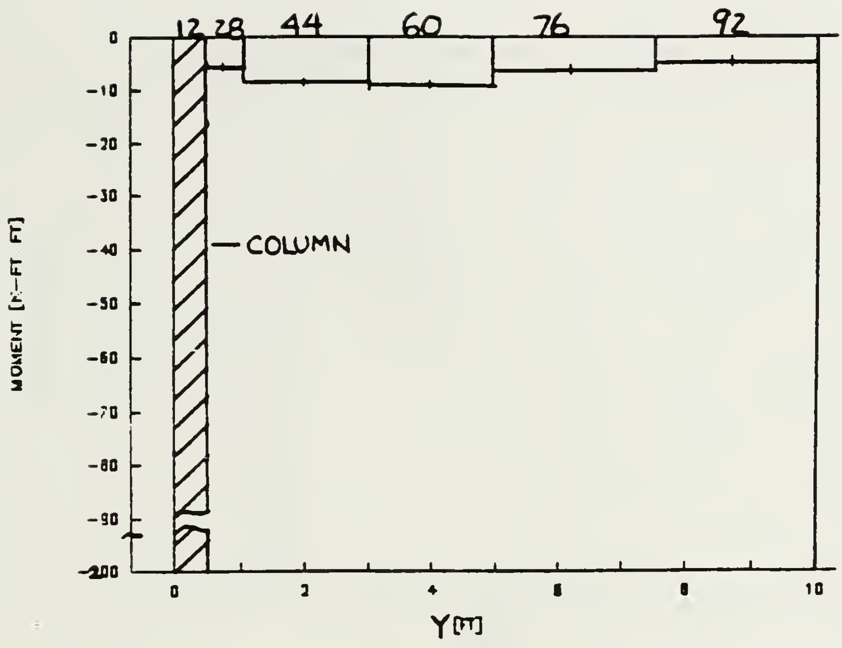
(see Figure 3.1 for element location)

Figure 6.12 Mx Moment for Plate Elements 8,24,40,56,72,& 88



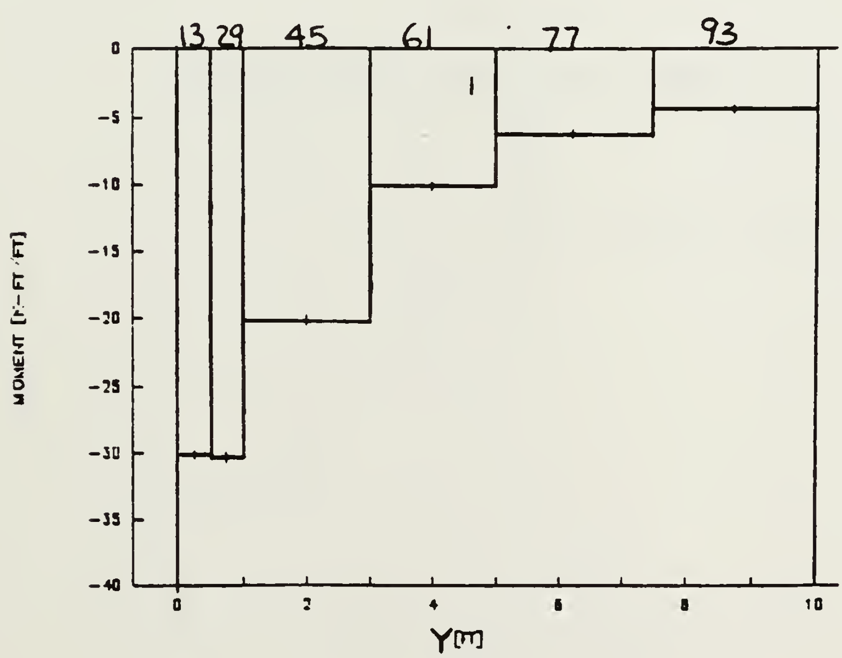
(see Figure 3.1 for element location)

Figure 6.13 Mx Moment for Plate Elements 9,25,41,57,73,& 89



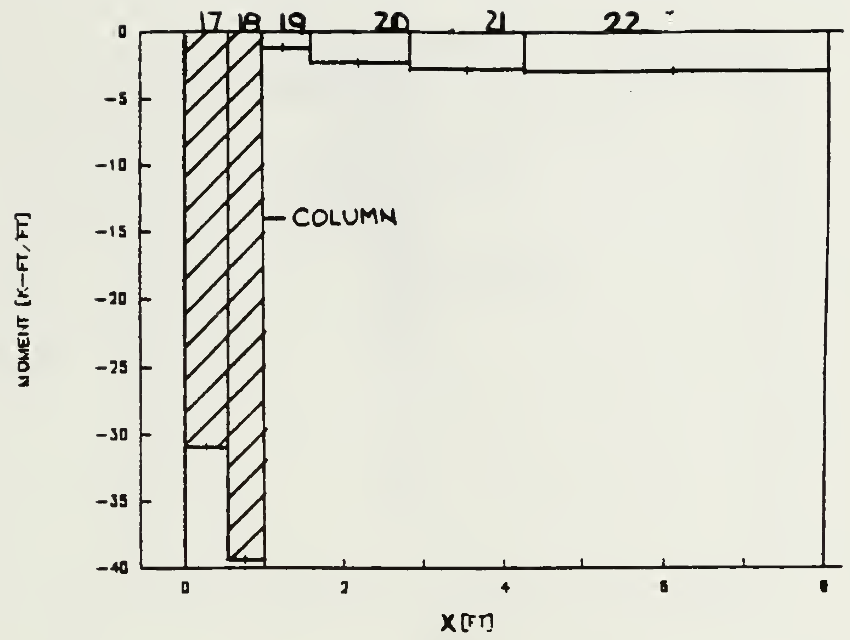
(see Figure 3.1 for element location)

Figure 6.14 Mx Moment for Plate Elements 12,28,44,60,76,& 92



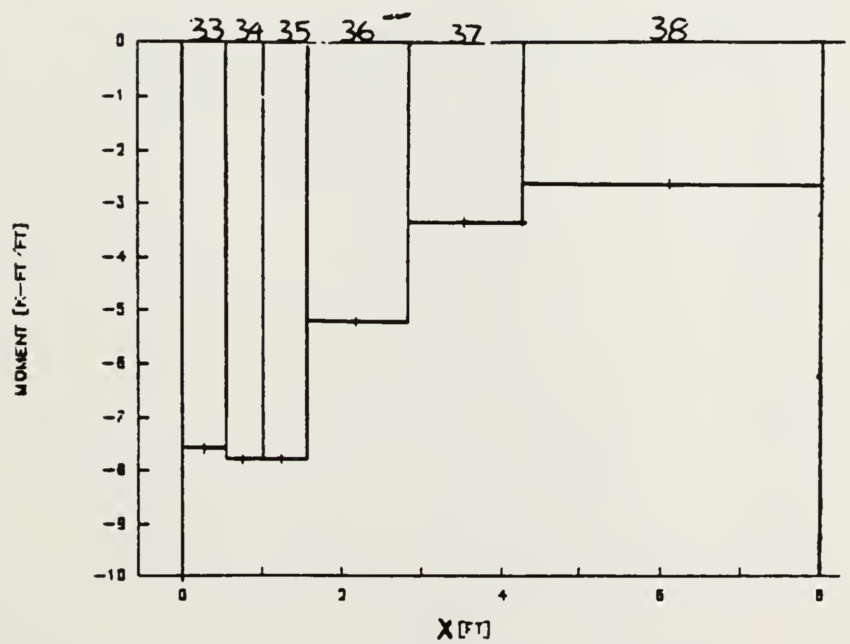
(see Figure 3.1 for element location)

Figure 6.15 Mx Moment for Plate Elements 13,29,45,61,77,& 93



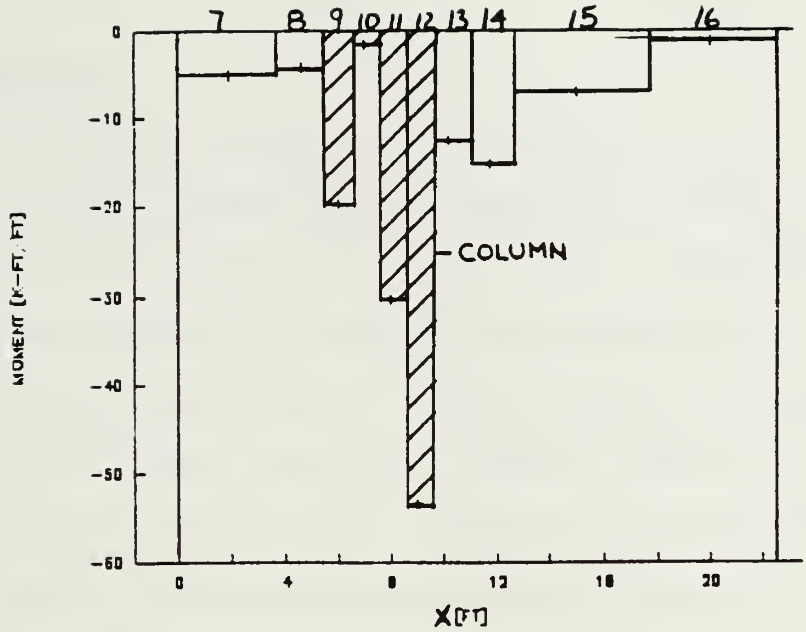
(see Figure 3.1 for element location)

Figure 6.16 My Moment for Plate Elements 17,18,19,20,21,& 22



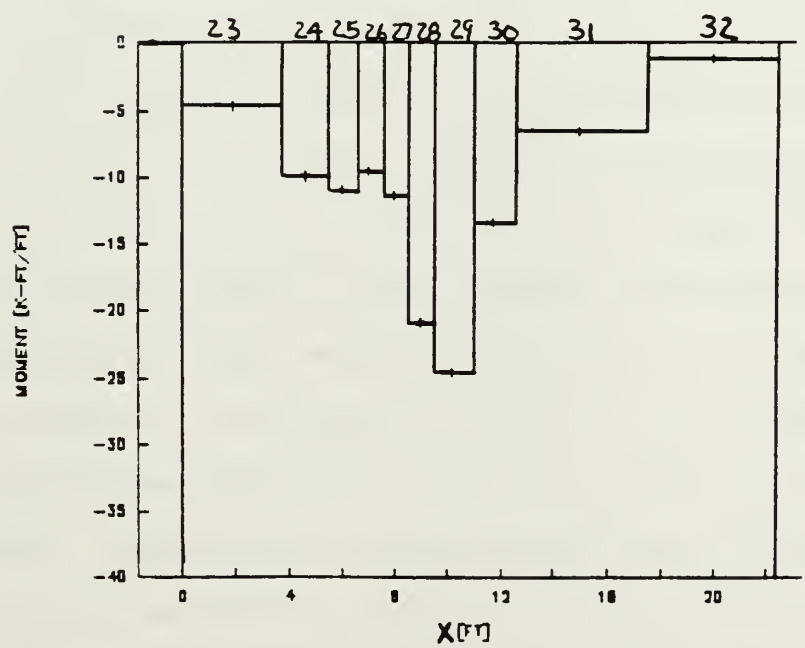
(see Figure 3.1 for element location)

Figure 6.17 My Moment for Plate Elements 33,34,35,36,37,& 38



(see Figure 3.1 for element location)

Figure 6.18 My Moment for Plate Elements 7,8,9,10,11,12,13,14,15,& 16



(see Figure 3.1 for element location)

Figure 6.19 My Moment for Plate Elements 23,24,25,26,27,28,29,30,31,& 32

6.3 Discussion

As previously stated, the very nature of the approaches used in designing the slab would lead to differences in the design moments. Since differences existed, the best way to compare the methods was to compare the clear span static moments from each method with the actual clear span static moment computed in accordance with the ACI Code. Tables 6.5 through 6.8 show the actual versus calculated static moment for each method for the various spans.

In all cases the Equivalent Frame Method static moment was the closest to the actual static moment. This was expected because the design approach was based on using the total static moment.

The static moments computed by the Strip Method were very close to the actual value with the exception of the longitudinal frame. The results for that frame indicated that both positive and negative moments were lower than for the other methods. Obviously this was an error; however, the source of the error is unknown. Perhaps the problem was in the way this particular span was modeled. The negative strip moments were found based on the assumption the reaction was distributed along the strong band. If, however, the reaction is assumed to be a point load acting at the center line of the strong band, the negative moments doubled. This

Table 6.5

CALCULATED VS. ACTUAL STATIC MOMENTS
TRANSVERSE FRAME -- EXTERIOR SPAN

METHOD	Mo (K-FT)	Mo CALC. (K-FT)	% DIFF.
EQUIVALENT FRAME	114.7	116.0	+1.0
FINITE ELEMENT	114.7	101.6	-11.5
STRIP	114.7	110.0	-4.1
YIELD LINE	114.7	129.2	+12.7

Table 6.6

CALCULATED VS. ACTUAL STATIC MOMENTS
TRANSVERSE FRAME -- INTERIOR SPAN

METHOD	Mo (K-FT)	Mo CALC. (K-FT)	% DIFF.
EQUIVALENT FRAME	496.2	496.6	+0.1
FINITE ELEMENT	496.2	470.0	-5.3
STRIP	496.2	496.0	-0.1
YIELD LINE	496.2	496.0	-0.1

Table 6.7

CALCULATED VS. ACTUAL STATIC MOMENTS
LONGITUDINAL FRAME

METHOD	Mo (K-FT)	Mo CALC. (K-FT)	% DIFF.
EQUIVALENT FRAME	301.0	301.5	+0.2
FINITE ELEMENT	301.0	283.0	-6.0
STRIP	301.0	228.0	-24.2
YIELD LINE	301.0	290.5	-3.5

Table 6.8

CALCULATED VS. ACTUAL STATIC MOMENTS
EDGE FRAME

METHOD	Mo (K-FT)	Mo CALC. (K-FT)	% DIFF.
EQUIVALENT FRAME	84.8	84.6	-0.2
FINITE ELEMENT	84.8	77.0	-9.2
STRIP	84.8	84.4	-0.5
YIELD LINE	84.8	100.0	+17.9

would account for some, but not all, of the error.

The Finite Element static moments were consistently less than the actual static moments by 5.3% to 11.5%. A slight under prediction of internal forces and moments is expected in a displacement based formulation of the Finite Element Method. Accuracy can be improved by decreasing the mesh size. The clear span total static moment was computed by plotting the moments at the center of each element and determining the clear span moment from the plot. Since the maximum positive and column face moments were read from the plot, an error in plotting or reading the moments at the critical sections would alter the resulting clear span moment.

The static moments computed by the Yield Line Method varied considerably from the actual static moments. The largest variations occurred in the exterior span of the transverse frame and in the edge frame. The computed static moment for the exterior span was 12.7% higher than the actual value. The high value was caused by the 25 K-FT moment used for design at the exterior column. Although the analysis was done under the assumption the exterior column could carry no moment, the CEB recommended reinforcing the column to carry a moment equal to one half the positive moment [1]. If this moment had been neglected in the static

moment calculation, the difference would have been only 1%.

The variation in the edge frame static moment was also due to design assumptions. During analysis, the exterior panel was designed as a whole, not broken into the edge and part of the longitudinal frames as in the Equivalent Frame Method. An average value of the clear span was used to compute the static moment for the panel. The moments were then distributed to the edge and longitudinal frames. Since the clear span for the edge frame was actually less than the clear span used to calculate the design moments, the calculated moments were higher. If the static moment for the exterior span were redistributed so an additional 14 K-FT went to the longitudinal frame, both values would have been within 1% of the actual value.

The distribution of negative moments between the column and middle strips was fairly consistent between methods. The Equivalent Frame and Yield Line Methods had a distribution of 75% column strip and 25% middle strip by design. The distribution for the Finite Element Method was approximately 70% column strip and 30% middle strip. This confirmed the distribution values used in the other methods were reasonable. Although there was more variation between frames, the

Strip Method distribution averaged about 70%.

The variation of positive moments was greater. For the Strip Method, almost all the positive moment was in the column strip. This resulted from the way the slab was modeled. The loading on most strips was such that each strip had a very small positive moment. Most of the positive moments were carried by the strong bands, which were in the column strips. The Finite Element Method results indicated, in general, that the middle strip carried 55% of the total positive moment elastically. It must be remembered, however, that the total static moment computed by the Finite Element Method was always lower than the actual static moment. The bending moment diagrams showed that perhaps, some moment was being lost in the column strip positive moment. If this were true, then the column strip positive moment should be increased, which would bring the distribution in line with the other methods.

The ratios of the average negative to positive moment was also an interesting comparison. The Finite Element Method moments were distributed such that the negative moments were approximately 1.47 times the positive moments for interior spans. The value was about 3 for the exterior span with the free edge. Based on the CEB's recommendations, the negative moment ratios used in the Yield Line Method were 1.5 for the

interior spans and 2.0 for the span with the free edge. The ratios computed using the Equivalent Frame were 1.2 and 3.0, respectively. The ratios computed by the Strip Method varied considerably for the interior spans. The transverse interior span had a ratio of 1.14, while the two other interior spans had values of about 2. The difference here can also be attributed to the way the slab was modeled.

Chapter 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

The results indicated the four methods chosen were acceptable for the design of the slab. Although variations in complexity and moments existed, each method would clearly produce a slab capable of resisting the applied loads. However, each method had its relative strengths and weaknesses. The following paragraphs will address these issues.

The Equivalent Frame Method done by hand was time consuming and relatively complex. Most of the effort for this design method was determining the member stiffnesses, fixed end moments, and carryover factors to be used in the moment distribution. Because of the large interior columns, the ratio of column length to span length was not printed in Table 13.1 of the ACI Code. Therefore, the values had to be extrapolated which induced some error. Without the use of the ACI Table, however, the task would have taken much longer. Once the moments were computed, interpolation was still required to determine the distribution of moments between the column and middle strips.

The Strip Method was relatively easy to use. The design of each strip was straightforward; however, some

moment distribution scheme was required to determine the carryover factors and relative stiffnesses. Therefore, any advantage over the Equivalent Frame Method was lost. Additionally, this method indicated the column strips should carry a large majority of the negative and positive moments. This would result in slabs with large amounts of steel in the column strips and only minimum steel in the middle strips. Since almost all of the middle strips in this study only required minimum steel, the result would have been a costlier design.

The Finite Element Method of design produced good results. Although the computed static moment for each span was less than the actual, this is attributed to the number of elements used and computing moments at the center of the element. Accuracy could be improved by using additional elements and computing moments at nodal points rather than averaging at the center. The low positive moments could be the result of using the larger elements in the positive moment regions. Although the computer input for the SAPIV program was rather lengthy, additional elements could have been added without much difficulty. These additional elements should be placed in the positive moment regions. Another problem was the computer output for each plate

element was the average moment per unit length. Therefore, the total moment in each element had to be computed by hand. This required much busy work which could have been handled by suitable pre- and post-processors. The strength of this approach was that it provided a theoretical distribution of the moments which was useful in comparing the assumptions used in the other design methods.

The Yield Line Method approach was by far the easiest method to use. The folding pattern assumed in each direction resulted in straightforward moment calculations. The CEB's recommendations for choosing the negative to positive moment ratios were in general agreement with the Finite Element results. The distribution of moments between the column and middle strip closely matched the distribution found using either the Finite Element or Equivalent Frame Methods. The CEB's recommendation for moment in the panel with the free edge was considerably low for this slab. All other methods indicated the negative moment in the exterior span should have been about 3 times the positive moment. CEB, however, recommended the value be limited to two. The result was a higher positive and lower negative moment than calculated by the other methods. The designer must also be careful to perform the checks for the complex yield line pattern and the local column

failure pattern. Although the complex yield pattern did not govern in any case, the local column yield pattern could have been a problem at the exterior column. The design moments at the column had to be increased slightly to ensure this pattern would not govern the design.

7.2 Recommendations

Based on this study, the Yield Line Method is recommended for the design of this type of slab. The method is by far the simplest method of design. However, good engineering judgment is required to ensure a serviceable slab. Recommendations are available in the literature to achieve this.

The simple Strip Method, as used in this study, does not seem well suited to column supported slabs because of the difficulty of selecting appropriate strong bands.

The Finite Element Method based on elastic analysis can be used to design slabs of arbitrary configuration. However input preparation and interpretation of results can be tedious without the aid of pre- and post-processors. Some care is also required in interpreting computed moments. In most cases, minimum reinforcement requirements will compensate for any

under-prediction of the bending moment. However, in some cases, it may be appropriate to adjust the computed moments based on the total static moment for a given span.

The Equivalent Frame Method, which is also based on elastic analysis, is tedious when performed by hand. However computer programs based on this method are available.

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Thesis
G375 Gibbs
c.1 Comparitive study of
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